Evaluation of Price-Sensitive Loads' Impacts on LMP and Market Power using LMP Decomposition

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Abstract: This paper presents a novel approach for evaluating impacts of price-sensitive loads on electricity price and market power. To accomplish this aim an analytical method along with agent-based computational economics are used. At first, Nash equilibrium is achieved by computational approach of Q-learning then based on the optimal bidding strategies of GenCos, which are figured out by Q-learning, ISO's social welfare maximization is restated considering demand side bidding. In this research, it was demonstrated that Locational Marginal Price (LMP) at each node of system can be decomposed into five components. The first constitutive part is a constant value for the respective bus, while the next two components are related to GenCos and the last two parts are associated to Load Serving Entities (LSEs). Market regulators can acquire valuable information from the proposed LMP decomposition. First, sensitivity of electricity price at each bus and Lerner index of GenCos to the bidding strategies and maximum price-sensitive demand of LSEs are revealed through weighting coefficients of the last two terms in the decomposed LMP. Moreover, the decomposition of LMP expresses contribution of LSEs to the electricity price. The simulation results on two test systems confirm the capability of the proposed approach.

Keywords: Electricity market, Locational marginal price, Market power, LMP decomposition, Price-sensitive load, Load Serving Entity (LSE).

1 Introduction

1.1 Motivation

Structure of electric power industry has been reformed by liberalization process in almost all over the world in the past three decades. Promoting competition and increasing efficiency are main goals of this restructuring [1]. In the deregulation regime electricity price has been the focus of all activities [2]. Thus, proper understanding of the electricity price behavior, which is derived by intersection of supply and demand, is essential for market regulators. Market power has been one of the key concerns of economists, which can significantly harm market efficiency. The likelihood of gaming the market is raised in the markets with low price-responsive demands, which is indeed one of the common features of electricity markets [3]. Therefore, using an effective technique for assessment and quantification of impacts of price-sensitive loads on electricity price and market power is of great importance for secure and economic operation of power systems.

1.2 Literature Review

The U.S. Department of Justice and Federal Trade Commission defined the seller's market power as "the ability to profitably maintain prices above competitive levels for a significant period of time" [4]. In order to detect and measure the market power a broad range of methods and indices including structural and behavioral indices as well as the various simulation approaches has been introduced and developed. The structural indices such as Herfindahl-Hirschman index (HHI), market share indices, residual supply index, pivotal supplier index and must run ratio are employed to detect the potential of the market power [5-9]. While on the contrary, the behavioral indices such as Lerner Index (LI), price-cost margin index (PCMI), and quantity modulated price index (QMPI) are used to check out the actual market power exercised [10-12].

Various research works on the market power analysis and strategic bidding behaviors have been ignored role of price-responsiveness of demands [12-16]. Demand response can be defined as "the changes in electric

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usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity" [17]. A wide range of demand response programs exists. Demand bidding can be considered as a subcategory from market based programs category of demand response [17]. Pre-specified matrices were used in [18-19] for describing changes in demands with respect to price variations. The difficulty in these approaches arises from the fact that pre-specified demand price-sensitivity matrices are hard to derive. Su and Kirschen [20] proposed a day-ahead market clearing tool which offers consumers the opportunity to reduce their energy costs by submitting a shifting bid. Influence of shifting the price responsive demand from periods of high price to periods of low price on congestion and locational marginal price is investigated in [21]. Wu in [22] investigates the impacts of price-based demand response on power system operation via network-constrained unit commitment model. However, despite the presented studies on the demand price responsiveness, no analytical models for evaluating the impacts of price-sensitive demand on the electricity price and market power based on the LMP structure can be found in the literature. Since analytical methods discover interrelationships and relative importance of different components that make up a phenomenon, these methods models can obtain deep insight into that phenomenon or system. Therefore, we have utilized an analytical approach.

1.3 The Proposed Approach and Contributions

This paper investigates impacts of price-sensitive loads on electricity price and market power of generation companies by decomposing and analyzing the locational marginal price (LMP) in a pool-based electricity market. To accomplish this aim an analytical method along with agent-based computational economics are used. At first step Nash equilibrium is achieved by computational approach of Q-learning (QL) then in the second step based on the optimal bidding strategies of GenCos which is computed by QL, ISO's social welfare maximization is restated considering demand side bidding. Therefore, in this study, demand is not inelastic anymore and load serving entities express their willingness to pay for demand through a linear bid function. Then, the optimization problem is solved using Lagrangian relaxation method and LMP at each bus is calculated. Afterwards LMP_n (LMP at node n) is manipulated and decomposed into five components. The first component is a constant value for each bus, which is independent from bidding strategies of GenCos and LSEs. The second and third components are associated to generating units include weighted summation of strategies of unbounded units (marginal units) and generated power of bounded generating units (units facing their generation caps), respectively. The fourth component of LMP is weighted sum of demanded power of fully dispatched LSEs and fifth part is weighted aggregation of bidding strategies of LSEs, which are not completely dispatched.

The presented decomposition obtains considerable information about impacts of price-sensitive loads on LMP and market power in Nash equilibrium (NE) of electricity market. Sensitivity of electricity price at each bus and Lerner index of GenCos to the bidding strategies and maximum demand of price-sensitive loads are indicated by weighting coefficients of the fourth and fifth term in decomposed LMP, respectively. Moreover, decomposition of LMP reveals contribution of each LSE to the electricity price at each bus. Therefore, the proposed approach can be employed as an efficient approach for assessment of influences of price-sensitive loads on market power and consequently enact proper policies for encouraging loads' responsiveness in order to mitigate market power. The simulation results on two test systems demonstrate the efficiency of the presented approach.

1.4 Paper Organization

The rest of this paper is organized as follows. Problem formulation is presented in section 2. Sections 3 and 4 include the proposed LMP decomposition and assessment of price-sensitive loads' impacts on market power, respectively. The simulation results for a 5-bus test system are presented in section 5. Finally, the paper is summarized and concluded in section 6.

2 Problem Formulation

2.1 Market Model

Let us consider a pool-based electricity market in which both generating companies and load service entities submit their hourly bids to an independent system operator (ISO). Another principal trait of the electricity markets is pricing mechanism. In our work, we focus on the closed auction with uniform pricing rule, which is the most commonly accepted structure of the spot electricity markets around the world. Assuming the quadratic form for generation cost function of GenCos, the marginal cost function will be in form of a linear increasing function. GenCos offer a linear supply function to the ISO in which the slope and/or intercept strategically changed regard to true supply function (marginal cost). In our work, we assume that GenCos only change their strategies by only adjusting the intercept value i.e. a_i, which is a rational and common assumption [23-24] as shown in Fig. 1. So, GenCos' supply function is expressed as follows.

$$\begin{aligned} &\text{bid}_i(Q_{Si}) = a_i + b_i Q_{Si} \\ &\text{where: } 0 \le Q_{Si} \le \overline{Q}_{Si} \end{aligned} \tag{1}$$

We assume that LSE j's demand is composed of a fixed component (Q^F_{Dj}) and a price-sensitive one (Q^S_{Dj}). Therefore, the demand of LSE j is $Q_{Dj} = Q^F_{Dj} + Q^S_{Dj}$. LSE j offers a linear inverse function for its price-sensitive demand over a known purchase interval:

$$\text{bid}_j(Q^S_{Dj}) = c_j - d_j Q^S_{Dj}$$
$$\text{where: } 0 \le Q^S_{Dj} \le Q^{S,max}_{Dj} \tag{2}$$

Also we assume LSEs adjust their bidding strategies by regulating the intercept of the line (2). Furthermore, in order to evaluate impacts of price-sensitivity of loads on LMP and market power the ratio R is defined as [25]:

$$R = \frac{Q^{S,max}_{Dj}}{Q^{S,max}_{Dj} + Q^F_{Dj}} \tag{3}$$

In which, the denominator is maximum potential total demand (MPTD). As illustrated in Fig. 2. by increasing R from zero (100% fixed demand case) to the value one (100% price-sensitive case) impacts of price-sensitivity is become clearer.

ISO receives the offers from the GenCos and LSEs then settles the market. ISO maximizes social welfare while matching supply and demand and satisfying transmission network constraints as expressed in Eq. (4).

$$\begin{aligned}
\text{Max} \quad J = & \sum_{j\in N_D}\left(c_j Q^S_{Dj} - 0.5 d_j {Q^S_{Dj}}^2\right) \\
& - \sum_{i\in N_S}\left(a_i Q_{Si} + 0.5 b_i {Q_{Si}}^2\right) \\
\text{s.t.} \quad & \\
& \sum_{j\in N_D} Q_{Dj} - \sum_{i\in N_S} Q_{Si} = 0 \quad \Leftrightarrow (\lambda) \\
& \underline{\alpha}_l \le \sum_{n\in N} \gamma_{l,n}\left(Q^n_S - Q^n_D\right) \le \overline{\alpha}_l \quad \Leftrightarrow (\Gamma^{\min}_l, \Gamma^{\max}_l) \;\; l \in L \\
& Q_{Si} \le \overline{Q}_{Si} \quad \Leftrightarrow (\mu_i) \\
& Q^S_{Dj} \le Q^{S,max}_{Dj} \quad \Leftrightarrow (\omega_j)
\end{aligned} \tag{4}$$

The Lagrangian relaxation method is employed to solve the optimization problem in (4). The corresponding Lagrangian formulation for the maximization problem (4) can be stated as,

$$\begin{aligned}
L = & \sum_{i\in N_S}\left(a_i Q_{Si} + 0.5 b_i {Q_{Si}}^2\right) - \sum_{j\in N_D}\left(c_j Q^S_{Dj} - 0.5 d_j {Q^S_{Dj}}^2\right) \\
& + \lambda\Big(\sum_{j\in N_D} Q^S_{Dj} + \sum_{j\in N_D} Q^F_{Dj} - \sum_{i\in N_S} Q_{Si} \Big) + \\
& \sum_{i\in N_S}\left(\mu_i \left(Q_{Si} - \overline{Q}_{Si}\right)\right) + \sum_{j\in N_D}\left(\omega_j \left(Q^S_{Dj} - Q^{S,max}_{Dj}\right)\right) + \\
& \sum_{l=1}^{L}\left(\Gamma^{\min}_l \left(\underline{\alpha}_l - \sum_{n\in N} \gamma_{l,n}\left(Q^n_S - Q^n_D\right)\right)\right) + \\
& \sum_{l=1}^{L}\left(\Gamma^{\max}_l \left(\sum_{n\in N} \gamma_{l,n}\left(Q^n_S - Q^n_D\right) - \overline{\alpha}_l\right)\right)
\end{aligned} \tag{5}$$

2.2 Modeling Power Supplier's Strategic Behavior

Supply side structure in electricity markets is usually oligopolistic. The oligopoly is strongly associated with mutual dependency between the market participants' behavior. The market participants learn how to react to competitors' behavior and market conditions in repeated games. Fundamentally, the learning ability plays an essential role in decision-making processes [26]. RL has been identified as an appropriate computational method to model electricity market participants' strategic behavior [27-29]. The RL problem is the learning problem for an agent on how to interact with its environment in order to achieve its goals. The agent and the environment interact in a sequence of discrete time-steps. Assume that S is a finite set of possible states of environment and A is a finite set of admissible actions, which the agent can take. At each time-step *t*, the agent senses the current state of the environment $s_t \in S$ and selects an action $a_t \in A$ accordingly. As a result of the agent's action, the state of the environment changes to the new state $s_{t+1} \in S$ and the agent receives an immediate reward r_{t+1}.

Watkins's QL algorithm [30], as a kind of model-free RL, is used to model the agents' learning behavior in the agent-based simulation. In the QL, for each admissible pair (s,a), a value function is defined as a Q-value. An agent attempts to find the optimal policy for each state to maximize the Q-value in the long-run. Proven in a self-play problem, without learning the model of environment, the QL is capable of determining the optimal policy by online estimation of its Q-value using the zero-order temporal difference method. Note that the convergence of QL to optimal policy is not guaranteed in multi-agent systems. After taking action a_t, the only available information for the QL is s_t, a_t, s_{t+1}, and r_{t+1}. The updating rule for $Q(s_t,a_t)$ is given by

$$\begin{aligned}
Q_{t+1}(s_t,a_t) &= Q_t(s_t,a_t) + \alpha \Delta Q(s_t,a_t) \\
\Delta Q(s_t,a_t) &= r_{t+1} + \gamma \max_{a'} Q(s_{t+1},a') - Q_t(s_t,a_t)
\end{aligned} \tag{6}$$

where α can be interpreted as how much the estimated Q–values are updated by new data. γ means how important the expected future reward is. In addition, the agent can use the ε parameter, known as ε-greedy strategy, to make a trade-off between exploitation and exploration.

To model the power supplier's strategic behavior, basic components of the QL are defined as follows.

1) State of environment: Keeping in mind the state of environment in the QL, the agent can use the memory to remember the experiences in different conditions of environment. The LMP is the main indicator of market conditions. Therefore, the state can be defined as a combination of LMP discretizations at all buses.

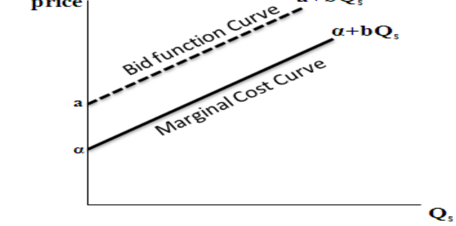

Fig. 1 Marginal cost and bid function curve.

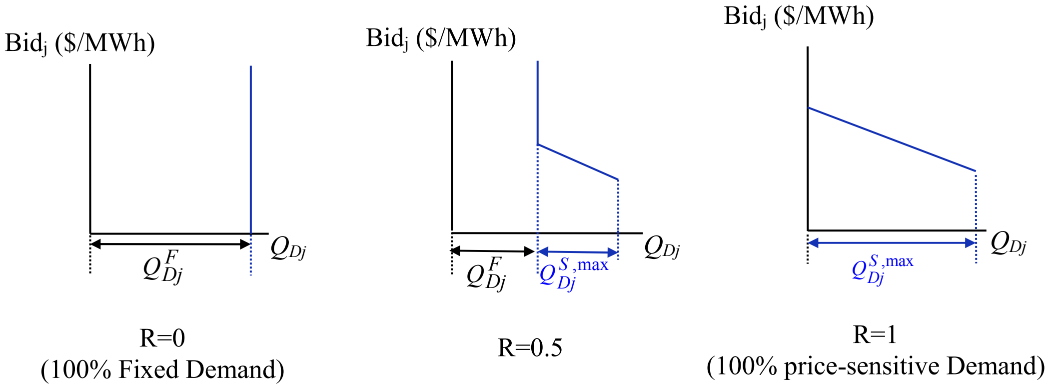

Fig. 2 Illustration of the R ratio construction for control of relative LSE demand-bid price sensitivity.

2) Agent's action: As stated earlier a power supplier may exercise market power through deviating intercept of supply function from the true corresponding coefficients of marginal cost. It is assumed the power supplier does not apply physical capacity withholding and offers as the production limits. Therefore intercept of supply function make one-dimensional action space.

3) Reinforcement signal: In the electricity market, the main objective of the offering problem is to maximize profit. Thus, the power supplier's profit gained in each stage of the game can be a proper reinforcement signal.

3 LMP Decomposition

Based on the market power definition there is a direct relationship between the price and market power. Therefore, in this study the locational marginal price (LMP) is decomposed then analyzed to evaluate impacts of price responsiveness of loads on market power.

From Karush-Kuhn-Tucker (KKT) conditions LMPs are obtained. For the simplicity and without loss of generality, it is assumed that at the market equilibrium point the directions of the power flow in transmission lines are already known so the value of power flow of lines are positive. Therefore, the lower limits of the lines' flow in (4) are relaxed i.e. $\Gamma_l^{\min}=0$ for all lines. Moreover, it is assumed that at the market equilibrium point, the power generated by units belong to $\overline{N}_S$ are limited to their upper capacity and LSEs belong to $\overline{N}_D$ are fully dispatched.

For the optimization problem (4) with the Lagrange equation described in (5) and based on the DC power flow, lemma 1 expresses the decomposition of LMP_n into five main components, derived from solving the KKT conditions for the Lagrange equation (5) at the market equilibrium point.

Lemma 1:

For the specified network topology, and based on the DC load flow, the LMP_n is obtained as follows:

$$\begin{aligned} LMP_n &= A_{0,n} + \sum_{i\in N_S-\overline{N}_S} A_{i,n} a_i + \sum_{i\in \overline{N}_S} A'_{i,n}\overline{Q}_{Si} \\ &\quad + \sum_{j\in N_D} A''_{j,n} Q_{Dj}^{S,\max} + \sum_{j\in N_D-\overline{N}_D} A'''_{j,n} c_j \end{aligned} \tag{7}$$

Proof:

The Lemma 1 is proved in two steps. In the first step, the KKT conditions for the optimization problem (4) at the market equilibrium point are analyzed. In the second step, by manipulating the results of the KKT conditions, the lemma 1 is proved.

Karush-Kuhn-Tucker conditions for (4) at the market equilibrium point are presented in Appendix A. Quantity of power generated by each unit and quantity of price-sensitive load dispatched are given in (8) and (9), respectively.

$$\begin{cases} Q_{Si} = \left(\lambda - a_i - \sum_{l\in L_{cong}} \Gamma_l^{\max}\gamma_{l,i}\right)\Big/ b_i & i\in N_S-\overline{N}_S \\ Q_{Si} = \overline{Q}_{Si} & i\in \overline{N}_S \end{cases} \tag{8}$$

$$\begin{cases} Q_{Dj}^{S} = \left(-\lambda + c_j + \sum_{l\in L_{cong}} \Gamma_l^{\max}\gamma_{l,i}\right)\Big/ d_j & j\in N_D-\overline{N}_D \\ Q_{Dj}^{S} = Q_{Dj}^{S,\max} & j\in \overline{N}_D \end{cases} \tag{9}$$

By substituting (8) and (9) in equality constraint, λ is given in (10).

$$\begin{aligned} \lambda &= \frac{\sum_{j\in N_D} Q_{Dj}^{F} + \sum_{j\in N_D} Q_{Dj}^{S,\max} - \sum_{i\in \overline{N}_S}\overline{Q}_{Si} + \sum_{i\in N_S-\overline{N}_S}\left(\frac{a_i}{b_i}\right) + \sum_{j\in N_D-\overline{N}_D}\left(\frac{c_j}{d_j}\right)}{C_l} \\ &\quad + \sum_{l\in L_{cong}}\left(\Gamma_l^{\max}\frac{\sum_{i\in N_S-\overline{N}_S}\left(\frac{\gamma_{l,i}}{b_i}\right) + \sum_{j\in N_D-\overline{N}_D}\left(\frac{\gamma_{l,j}}{d_j}\right)}{C_l}\right) \end{aligned} \tag{10}$$

where $C_1 = \sum_{i\in N_S-\overline{N}_S} \frac{1}{b_i} + \sum_{j\in N_D-\overline{N}_D} \frac{1}{d_j}$.

The electricity price at each bus(LMP) is given in (11), which is the marginal cost of the marginal unit at the respective bus [31].

$$LMP_n = \lambda - \sum_{l\in L_{cong}} \Gamma_l^{max}\gamma_{l,n} \tag{11}$$

If there is no congestion in the network, λ is the Market Clearing Price (MCP) and equals the first term in the right-hand side of (10). From KKT condition associated to binding inequality constraints and after some manipulation which are given in Appendix A, relationship among Γ_l^{max}'s, a_i's, $\overline{Q}_{Si}$, $Q_{Dj}^{S,max}$ and c_j's is obtained as (12).

$$\sum_{m\in N_S-\overline{N}_S}\left(\frac{\sum_{i\in N_S-\overline{N}_S}\left(\frac{\gamma_{l,i}}{b_i}\right)}{C_1\ b_m} - \frac{\gamma_{l,m}}{b_m} + \frac{\sum_{r\in N_D-\overline{N}_D}\frac{\gamma_{l,r}}{d_r}}{C_1\ b_m}\right)a_m + \sum_{k\in L_{cong}}\Gamma_k^{max}$$
$$\times\Bigg(\sum_{m\in N_S-\overline{N}_S}\left(\frac{\sum_{i\in N_S-\overline{N}_S}\left(\frac{\gamma_{l,m}\gamma_{k,i}}{b_i}\right)}{C_1 b_m} - \frac{\gamma_{l,m}\gamma_{k,m}}{b_m} + \frac{\sum_{j\in N_D-\overline{N}_D}\frac{\gamma_{l,m}\gamma_{k,j}}{d_j}}{C_1 b_m}\right) + \tag{12}$$
$$\sum_{r\in N_D-\overline{N}_D}\left(\frac{\sum_{i\in N_S-\overline{N}_S}\left(\frac{\gamma_{l,r}\gamma_{k,i}}{b_i}\right)}{C_1 d_r} - \frac{\gamma_{l,r}\gamma_{k,r}}{d_r} + \frac{\sum_{j\in N_D-\overline{N}_D}\frac{\gamma_{l,r}\gamma_{k,j}}{d_j}}{C_1 d_r}\right)\Bigg) =$$
$$\overline{\alpha}_l + \sum_{j\in N_D}\gamma_{l,j}Q_{Dj}^F + \sum_{j\in N_D}\gamma_{l,j}Q_{Dj}^{S,max} - \sum_{i\in \overline{N}_S}\gamma_{l,i}\overline{Q}_{Si} + \sum_{r\in N_D-\overline{N}_D}\gamma_{l,r}\frac{c_r}{d_r}$$
$$-\left(\sum_{m\in N_S-\overline{N}_S}\frac{\gamma_{l,m}}{b_m} + \sum_{r\in N_D-\overline{N}_D}\frac{\gamma_{l,r}}{d_r}\right)D_1$$

where $D_1 = \left(\frac{1}{C_1}\right)\left(\sum_{j\in N_D}Q_{Dj}^F + \sum_{j\in N_D}Q_{Dj}^{S,max} - \sum_{i\in \overline{N}_S}\overline{Q}_{Si} + \sum_{j\in N_D-\overline{N}_D}\left(\frac{c_j}{d_j}\right)\right)$.

Equation (13) is the vector form of (12) and shows there is a linear relationship between Γ_l^{max}'s, GenCos strategies a_i's, $\overline{Q}_{Si}$, price-sensitive loads' maximum demand and price-sensitive load strategies c_j's.

$$\begin{aligned}&\alpha_{L_{cong}\times(N_S-\overline{N}_S)}a + \beta_{L_{cong}\times L_{cong}}\Gamma^{max} = \\ &C - D_{L_{cong}\times\overline{N}_S}\overline{Q}_S + E_{L_{cong}\times\overline{N}_D}\overline{Q}_D^S + F_{L_{cong}\times(N_D-\overline{N}_D)}c\end{aligned} \tag{13}$$

where α, β, C, D, E and F are explained in Appendix A in (A5). Therefore relationship between Γ_l^{max}'s, a_i's, $\overline{Q}_s$, $\overline{Q}_D^S$ and c_j's is :

$$\begin{aligned}&\Gamma^{max} = \beta^{-1}\times C - \beta^{-1}\times\alpha\times a - \beta^{-1}\times D\times\overline{Q}_s + \\ &\beta^{-1}\times E\times\overline{Q}_D^S + \beta^{-1}\times F\times c\end{aligned} \tag{14}$$

in which, a is the vector of strategies of the GenCos, which contributing to the price discovery process, c is the vector of bidding strategies of the LSEs which are not fully dispatched, $\overline{Q}_s$ is the vector of maximum generation of the units, bound to their maximum generations and $\overline{Q}_D^S$ is the vector of price-sensitive loads' maximum demand of LSEs that are completely dispatched. Therefore, by substituting λ from Eq. (10) into Eq. (11), the following equations are obtained:

$$\begin{aligned}LMP_n = &\frac{\sum_{j\in N_D}Q_{Dj}^F + \sum_{j\in N_D}Q_{Dj}^{S,max} - \sum_{i\in \overline{N}_S}\overline{Q}_{Si}}{C_1} + \\ &\frac{\sum_{i\in N_S-\overline{N}_S}\left(\frac{a_i}{b_i}\right) + \sum_{j\in N_D-\overline{N}_D}\left(\frac{c_j}{d_j}\right)}{C_1} + \\ &\sum_{l\in L_{cong}}\left(\Gamma_l^{max}\frac{\sum_{i\in N_S-\overline{N}_S}\left(\frac{\gamma_{l,i}}{b_i}\right) + \sum_{j\in N_D-\overline{N}_D}\left(\frac{\gamma_{l,j}}{d_j}\right)}{C_1} - \Gamma_l^{max}\gamma_{l,n}\right)\end{aligned} \tag{15}$$

The LMP_n in (15) can be represented in the vector form in (16),

$$\begin{aligned}LMP_n = &C_2 + C_3\times\overline{Q}_s + C_4\times\overline{Q}_D^S + G\times c \\ &+ A\times a + B_n\times\Gamma^{max}\end{aligned} \tag{16}$$

in which C_2, C_3, C_4, G, A and B_n are presented in Appendix A in (A6). By replacing Γ_l^{max} from Eq. (14) in Eq. (16) LMP_n is obtained:

$$\begin{aligned}&LMP_n = C_2 + C_3\times\overline{Q}_s + C_4\times\overline{Q}_D^S + G\times c + A\times a + \\ &B_n\times\beta^{-1}\left(C - \alpha\times a - D\times\overline{Q}_s + E\times\overline{Q}_D^S + F\times c\right)\end{aligned}$$

$$LMP_n = A_{0,n} + A_n\times a + A'_n\times\overline{Q}_s + A''_n\times\overline{Q}_D^S + A'''_n\times c \Rightarrow \tag{17}$$

$$\begin{aligned}LMP_n = &A_{0,n} + \sum_{i\in N_S-\overline{N}_S}A_{i,n}a_i + \sum_{i\in\overline{N}_S}A'_{i,n}\overline{Q}_{Si} + \\ &\sum_{j\in N_D}A''_{j,n}Q_{Dj}^{S,max} + \sum_{j\in N_D-\overline{N}_D}A'''_{j,n}c_j\end{aligned}$$

where

$$\begin{cases}A_{0,n} = C_2 + B_n\times\beta^{-1}\times C \\ A_n = A - B_n\times\beta^{-1}\times\alpha \\ A'_n = C_3 - B_n\times\beta^{-1}\times D \\ A''_n = C_4 + B_n\times\beta^{-1}\times E \\ A'''_n = G + B_n\times\beta^{-1}\times F\end{cases}$$

Thus, the Lemma 1 is proved.

4 Assessment of Price-Sensitive Loads' Impacts on Market Power

Based on the LMP decomposition expressed by lemma 1, impacts of price-sensitive loads on LMP and consequently on the suppliers' market power can be evaluated. In addition, contribution of generating units in the electricity price at each bus can be identified using a similar approach as in [16]. According to (7) in lemma 1, the LSEs have been classified into two groups. The first group consists of LSEs that belong to $\overline{N_D}$, which are willing to pay more so they are fully dispatched in market equilibrium. The second group includes the LSEs that belong to $N_D - \overline{N_D}$, which their bids was lower so they did not fully dispatched. Electricity price at equilibrium point is directly influenced by bidding strategies of second group of LSEs. Based on the proposed classifications, some critical points can be derived from (7),

1- Term $A'''_{j,n} c_j$ represents the contribution of LSE j to the electricity price at bus n. Furthermore, according to (18), $A'''_{j,n}$ indicates the variation of the electricity price at bus n according to the variation of the bidding strategy of LSE j.

$$LMP_n = A_{0,n} + \sum_{i \in N_S - \overline{N_S}} A_{i,n} a_i + \sum_{i \in \overline{N_S}} A'_{i,n} \overline{Q}_{Si} + \sum_{j \in \overline{N_D}} A''_{j,n} Q^{S,max}_{Dj} + \sum_{j \in N_D - \overline{N_D}} A'''_{j,n} c_j \tag{18}$$

$$\Rightarrow \Delta LMP_n = A'''_{j,n} \times \Delta c_j \Rightarrow A'''_{j,n} = \frac{\Delta LMP_n}{\Delta c_j}$$

Thus, significant values for coefficient $A'''_{j,n}$ demonstrate high sensitivity of electricity price at bus n to the bidding strategy of LSE j. Therefore, knowing the values of $A'''_{j,n}$ and $A'''_{j,n} c_j$ at market equilibrium, system operator can easily identify the impacts of LSE j on market power at each bus of the power grid. In the case that congestion occurrences in the network, the coefficients $A'''_{j,n}$ can be either positive or negative. It means increasing in the bidding strategy of LSE j i.e. c_j does not necessarily result in increasing of LMP_n. Quite contrary, this may lead to price decreasing at bus n.

2- The value of $A''_{j,n} Q^{S,max}_{Dj}$ represents the contribution of LSE j which is fully dispatched in electricity price at bus n. Therefore large value of $A''_{j,n} Q^{S,max}_{Dj}$ indicates high sensitivity of the electricity price at bus n to the increasing of demand by LSE j. It must be noted that if congestion happens in the network, then the coefficients $A''_{j,n}$ can be either positive or negative. It means increasing in the maximum quantity demanded by LSE j does not necessarily result in increasing of LMP_n. Quite contrary, this may lead to price decreasing at bus n.

3- In order to evaluate impacts of price-sensitive loads on market power, sensitivity of Lerner index (LI) as a classic index for measuring market power to bidding strategy of LSEs had been calculated. Sensitivity of LI associated to unit *i* which is located at bus n to the bidding strategy of LSE j calculated as below:

$$\begin{cases} \dfrac{\Delta LI_i}{\Delta c_j} = \dfrac{-(a_i - \alpha_i) A'''_{j,n}}{LMP_n^2} & i \in N_S - \overline{N_S} \\ \dfrac{\Delta LI_i}{\Delta c_j} = \dfrac{-(\alpha_i + b_i \overline{Q_{Si}}) A'''_{j,n}}{LMP_n^2} & i \in \overline{N_S} \end{cases} \tag{19}$$

Thus, significant values for coefficient $A'''_{j,n}$ demonstrate high sensitivity of LI of unit i at bus n to the bidding strategy of LSE j. Therefore, knowing the values of $A'''_{j,n}$ market equilibrium, system operator can easily identify the impacts of LSE j on market power of each generating unit in power grid.

Based on the presented discussions, coefficients $A''_{j,n}$ and $A'''_{j,n}$ can be employed for evaluation impacts of price-sensitive loads on LMP and market power at market equilibrium.

5 Numerical Examples

Three case studies are presented in this section to illustrate the application of the proposed method. In the first two case studies a 5-bus transmission grid is used which is taken from ISO-NE/PJM training manuals. The IEEE 30-bus test system is considered in the third case study. The topology of the 5-bus test system is shown in Fig. 3. Details of the Line capacities, reactance levels, and generators' cost data of this test system are adopted from [32]. In the second case study, upper limit of flow of line1 connecting bus1 to bus2 decreased from 250 MW to 200 MW so possibility of congestion is acquired.

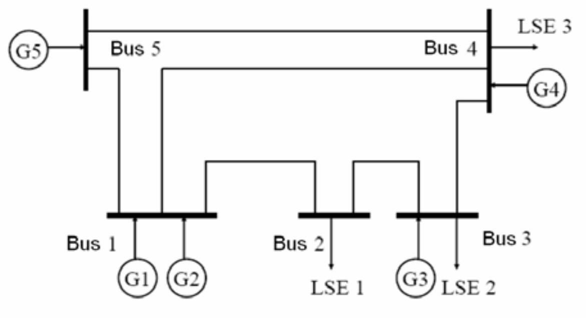

Fig. 3 5-bus test system.

Table 1 Generation and Load data for 5-bus test system.

Unit ID	1	2	3	4	5
At Node	1	1	3	4	5
Unit Size (MW)	110	100	520	200	600
a ($/MWh)	14	15	25	30	10
b ($/MW^{2}h)	.01	.012	.02	.024	.014

LSE ID	1	2	3
At Node	2	3	4
Maximum Potential Total Demand (MW	201.0	172.3	143.6
c ($/MWh)	35	40	28
d ($/MW^{2}h)	.18	.08	.12

Table 2 Q-learning results for market equilibrium in case 1.

R	a_i					Q_{Si}					LMP	Q_{Dj}^{S}			$A_{0,n}$
	i=1	*i=2*	*i=3*	*i=4*	*i=5*	*i=1*	*i=2*	*i=3*	*i=4*	*i=5*		*j=1*	*j=2*	*j=3*	
0	14	15	25	30	20.7	110	100	-	-	306.81	24.9953	-	-	-	7.2353
0.25	14	15	25	30	20.7	110	100	-	-	306.60	24.9924	50.04	43.07	35.89	5.0715
0.5	14	15	25	30	21.8	110	100	-	-	226.58	24.9721	50.14	86.14	41.90	3.0486
0.75	14	15	25	30	21.7	110	100	-	-	155.12	23.8716	55.64	129.20	51.07	1.5243

Table 3 Weighting coefficients and price components of the GenCos in case 1.

R	$A_{5,n}$	$A'_{1,n}$	$A'_{2,n}$	$A_{5,n}a_5$	$A'_{1,n}\overline{Q}_{S1}$	$A'_{2,n}\overline{Q}_{S2}$
0	1	- 0.014	- 0.014	20.7	-1.5400	-1.4000
0.25	0.9346	- 0.0131	- 0.0131	19.3458	-1.4393	-1.3084
0.5	0.8427	- 0.0118	- 0.0118	18.3708	-1.2978	-1.1798
0.75	0.8427	- 0.0118	- 0.0118	18.2865	-1.2978	-1.1798

Table 4 Weighting coefficients and price components of the LSEs in case 1.

R	$A''_{2,n}$	$A''_{3,n}$	$A'''_{1,n}$	$A'''_{3,n}$	$A''_{2,n}Q_{D2}^{S,\max}$	$A''_{3,n}Q_{D3}^{S,\max}$	$A'''_{1,n}c_1$	$A'''_{3,n}c_3$
0	-	-	-	-	-	-	-	-
0.25	0.0131	0.0131	0.0654	-	0.5635	0.4696	2.2897	-
0.5	0.0118	-	0.0590	0.0983	1.0162	-	2.0646	2.9494
0.75	0.0118	-	0.0590	0.0983	1.5243	-	2.0646	2.9494

5.1 Case Study 1:

The cost and capacity information of generation units and load data are presented in Table 1. It should be noted that for all simulations maximum potential total demand is considered constant. The QL parameters α, γ, and ε are 0.9, 0.1, and 0.1, respectively, these parameters are chosen based on previous efforts, in a way that a reasonable trade-off between exploitation and exploration is achieved. At first, Q-value is generated randomly based on uniform distribution. Afterward, we run the simulation with the assumed parameters for 10000 iterations. In order to evaluate impacts of price-sensitive loads on LMP and market power four values for parameter *R* is considered (0, 0.25, 0.5 and 0.75). There was no congestion in the transmission grid and therefore electricity price at all buses is the same. For each value of R market equilibrium was calculated using QL. Market simulation results for this case with various values for R are presented in Table 2. Table 3 shows the decomposition results for generating units and their impacts on electricity price. Table 4 demonstrates the decomposition results for LSEs and their influences on LMP. Based on the presented results in Tables 2, 3 and 4 the following remarks can be made.

- Expensive generating units (units 3 and 4) were limited to their minimum generations while generating units 1-2 were bound by their generation caps. Therefore, unit 5 was the only marginal unit (Table 2).
- As *R* was increased and consequently price-sensitivity of loads was raised, electricity price was decreased. Moreover, in Table 3 by increasing *R* influence of unit 5 on the electricity was decreased (coefficient $A_{i,n}$ for marginal unit was decreased). Therefore, market power of marginal unit was decreased as price-sensitivity of loads was increased.
- The bidding strategy of LSE 3 (at node 4) have the largest impact on electricity price (case R=0.5

and 0.75). For instance, if LSE 3 in case R=0.5 decreases its bid, i.e. c_j by 1 \$/MWh from 30 to 29 and the network be re-dispatched, the electricity price experiences a decrease of 0.0983 \$/MWh and reaches to 24.8738, which is compatible with (18) and anticipated in Table 4.

- If the network is not congested, then the coefficients $A''_{j,n}$ are always positive; that is if any LSE with tendency to pay more, which is fully dispatched, had larger demand then it could cause increase in the electricity price (Table 4).

5.2 Case Study 2:

In the second case study, as mentioned earlier only upper limit of flow of line1 which links bus1 to bus2 is decreased from 250 MW to 200 MW so possibility of congestion is achieved. The QL parameters and conditions are the same as the case1. Again, for parameter R four values are considered. For each value of *R* market equilibrium was calculated using QL. Simulation results for this case with various values for R are presented in Table 5. Tables 6 and 7 are showing the decomposition results for GenCos and LSEs respectively. As expected, when congestion occurs in network number of marginal units increases, which can be seen in Table 5. Based on the results presented in Tables 6 and 7 following remarks can be made.

- If congestion happens in the grid, then coefficients $A''_{j,n}$ are not always positive. This means increasing the maximum demand by LSE with tendency to pay more, does not necessarily results in increase of electricity price. For instance, in case R=0.5 if LSE 2 at bus 3 increases its amount of maximum price-sensitive demand i.e. $Q^{S,max}_{Dj}$ by 1 MW and re-dispatch the network, the electricity price at bus 1 decreases by 0.0012 \$/MWh, while LMP at bus 2 increases by 0.0213 \$/MWh. It should be noted that this price variation could be predicted by using the values of $A''_{j,n}$ in Table 7.
- The coefficients $A'''_{j,n}$ express the ability of LSE j to affect electricity price at bus n. For instance, electricity prices at bus 1 and bus 2 are highly sensitive to the bidding strategy of LSE 1 at bus 2. It is interesting to note that by increase of bid of LSE 1, the electricity price at bus 2 increases while LMP reduces at bus 1. For instance, in caser=0.5 if LSE 1 increases its bidding strategy by 1 \$/MWh, then the price at bus 2 increases by 0.1374 \$/MWh, reaching 26.1199 \$/MWh, i.e. 0.53% variation. On the other hand, under this condition, the LMP_1 decreases by 0.0252 \$/MWh to 25.6466 \$/MWh, i.e. -0.10% variation which is compatible with Table 7.

5.3 Case Study 3:

Further tests were performed using the IEEE 30-bus system. This test system is composed of 6 generators and 20 consumers (LSEs), as shown in Fig. 4. Details of the Line capacities, reactance levels are adopted from [33]. The cost and capacity information of generating units and load data are presented in Appendix B. The QL parameters and conditions are the same as previous cases. Tables 8 and 9 are demonstrating simulation results for this case with various values regards to parameter R. It must be mentioned that LMP values are presented only for nodes with generating units. Decomposition results for GenCos and LSEs are presented in Tables 10, 11 and 12, respectively. According to the results presented in Tables 8-12 following remarks can be made.

- As it is shown in Table 8 by increasing R and consequently price-sensitivity of loads, congestion in network was decreased. That is only for R=0 and R=0.25 network was congested.
- As mentioned earlier when congestion occurs coefficients $A''_{j,n}$ are not always positive. For instance, in case R=0.25 an increment in maximum price-sensitive demand i.e. $Q^{S,max}_{Dj}$ of LSE 12 by 1 MW, causes an increment of 0.0811 \$/MWh in LMP at bus 2, while LMP at bus 13 decreases by 0.2023 \$/MWh. It should be noted that this price variation could be predicted by using the values of $A''_{j,n}$ in Table 11.
- Market regulators can predict the ability of LSEs in manipulating electricity price and consequently market power of generating units by means of their bidding strategies through coefficients $A'''_{j,n}$. For instance, in case R=0.25 if LSE 13 increases its bidding strategy by 1 \$/MWh, then the price at bus 22 increases by 0.0566 \$/MWh, reaching 32.9566 \$/MWh, i.e. 0.17% variation. On the other hand, under the same assumption, the electricity price at bus 13 reduceds by 0.0402 \$/MWh to 31.0998 \$/MWh, i.e. -0.13% variation which is compatible with Table 12.

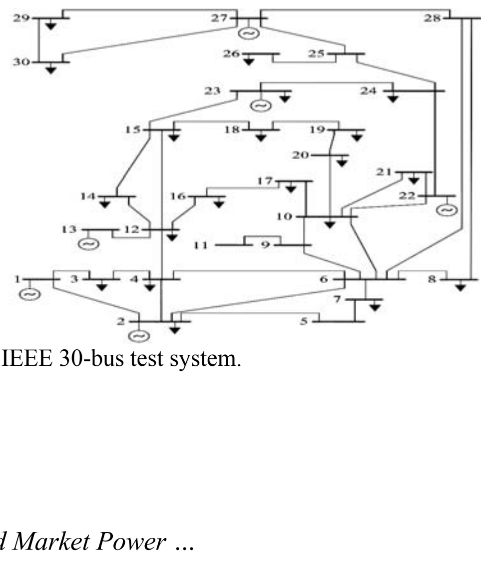

Fig. 4 IEEE 30-bus test system.

Table 5 Q-learning results for market equilibrium in case 2.

R	a_i					Q_{Si}					LMP					Q^S_{Dj}		
	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$j=1$	$j=2$	$j=3$
0	14	15	100	30	40	110	100	26.60	200	80.20	37.107	115.358	100.532	59.761	41.123	-	-	-
0.25	14	15	29.5	30	27	110	100	43.41	-	199.90	29.760	30.511	30.368	29.977	29.799	22.45	43.07	0.19
0.5	14	15	25.75	30	22.8	110	100	8.68	-	206.26	25.672	25.983	25.924	25.762	25.688	45.57	86.52	51.99
0.75	14	15	25	30	21.7	110	100	-	-	155.12	23.872	23.872	23.872	23.872	23.872	55.64	129.20	51.07

Table 6 Weighting coefficients and price components of the GenCos in case 2.

R	n	$A_{0,n}$	$A_{3,n}$	$A_{5,n}$	$A'_{1,n}$	$A'_{2,n}$	$A'_{4,n}$	$A_{3,n}a_3$	$A_{5,n}a_5$	$A'_{1,n}\overline{Q}_{S1}$	$A'_{2,n}\overline{Q}_{S2}$	$A'_{4,n}\overline{Q}_{S4}$
0	1	6.4996	-0.0676	1.0676	-0.0160	-0.0160	-0.0098	-6.7604	42.7042	-1.7653	-1.6048	-1.9666
	2	0.3351	1.2496	-0.2496	0.0054	0.0054	-0.0054	124.9554	-9.9822	0.5961	0.5419	-1.0885
	3	1.5030	1	0	0.0014	0.0014	-0.0063	100	0	0.1487	0.1352	-1.2549
	4	4.7149	0.3137	0.6863	-0.0098	-0.0098	-0.0086	31.3725	27.4512	-1.0817	-0.9833	-1.7124
	5	6.1832	0	1	-0.0149	-0.0149	-0.0096	0	40	-1.6441	-1.4946	-1.9216
0.25	1	6.1134	-0.0605	1.0081	-0.0151	-0.0151	-	-1.7843	27.2187	-1.6664	-1.5149	-
	2	-2.3401	1.0664	-0.2403	0.0050	0.0050	-	31.4574	-6.4885	0.5537	0.5034	-
	3	-0.7385	0.8529	-0.0038	0.012	0.012	-	25.1593	-0.1022	0.1331	0.1210	-
	4	3.6660	0.2657	0.6467	-0.0093	-0.0093	-	7.8395	17.4602	-1.0237	-0.9306	-
	5	5.6795	-0.0026	0.9440	-0.0141	-0.0141	-	-0.0781	25.4887	-1.5525	-1.4113	-
0.5	1	6.0844	-0.0605	1.0081	-0.0151	-0.0151	-	-1.5575	22.9847	-1.6664	-1.5149	-
	2	-4.7970	1.0664	-0.2403	0.0050	0.0050	-	27.4586	-5.4792	0.5537	0.5034	-
	3	-2.7354	0.8529	-0.0038	0.0012	0.0012	-	21.9611	-0.0863	0.1331	0.1210	-
	4	2.9341	0.2657	0.6467	-0.0093	-0.0093	-	6.8429	14.7442	-1.0237	-0.9306	-
	5	5.5259	-0.0026	0.9440	-0.0141	-0.0141	-	-0.0682	21.5238	-1.5525	-1.4113	-
0.75	all	1.5234	-	0.8427	-0.0118	-0.0118	-	-	18.2865	-1.2978	-1.1798	-

Table 7 Weighting coefficients and price components of the LSEs in case 2.

R	n	$A''_{2,n}$	$A'''_{1,n}$	$A'''_{3,n}$	$A''_{2,n}Q^{S,\max}_{D2}$	$A'''_{1,n}c_1$	$A'''_{3,n}c_3$
0.25	1	-0.0012	-0.0252	0.0776	-0.0521	-0.8809	2.3265
	2	0.0213	0.1374	0.0365	0.9185	4.8106	1.0955
	3	0.0171	0.1066	0.0443	0.7346	3.7322	1.3287
	4	0.0053	0.0219	0.0657	0.2289	0.7668	1.9701
	5	-0.0001	-0.0168	0.0754	-0.0023	-0.5888	2.2634
0.5	1	-0.0012	-0.0252	0.0776	-0.1042	-0.8809	2.3265
	2	0.0213	0.1374	0.0365	1.8370	4.8106	1.0955
	3	0.0171	0.1066	0.0443	1.4692	3.7322	1.3287
	4	0.0053	0.0219	0.0657	0.4578	0.7668	1.9701
	5	-0.0001	-0.0168	0.0754	-0.0046	-0.5888	2.2634
0.75	all	0.0118	0.059	0.0983	1.5243	2.0646	2.9494

Table 8 Market equilibrium results for case study 3.

R	***a_i***						***Q_{Si}***						**LMP_n**					
	i=1	*i=2*	*i=3*	*i=4*	*i=5*	*i=6*	*i=1*	*i=2*	*i=3*	*i=4*	*i=5*	*i=6*	*n=1*	*n=2*	*n=13*	*n=22*	*n=23*	*n=27*
0	20.70	23.50	26.87	36.30	25.85	29.60	66.47	69.41	15.67	28.38	20.10	32.01	37.317	37.408	30.009	41.975	29.870	37.603
0.25	19.80	22.50	26.87	25.52	25.30	22.00	49.67	48.65	21.32	36.92	29.10	41.04	32.217	32.231	31.139	32.904	31.119	32.259
0.5	19.80	21.50	25.75	23.65	23.10	19.20	40.08	41.60	20.35	30.85	33.60	42.48	29.819	29.819	29.819	29.819	29.819	29.819
0.75	18.90	21.30	25.37	23.10	23.10	18.00	35.23	31.03	11.66	23.03	23.03	38.83	27.707	27.707	27.707	27.707	27.707	27.707

Table 9 Q-learning results for LSEs in market equilibrium for case study 3.

R	Q^{S}_{Dj}																			
	j=1	*j=2*	*j=3*	*j=4*	*j=5*	*j=6*	*j=7*	*j=8*	*j=9*	*j=10*	*j=11*	*j=12*	*j=13*	*j=14*	*j=15*	*j=16*	*j=17*	*j=18*	*j=19*	*j=20*
0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.25	2.40	2.36	2.67	2.70	3.00	1.62	2.73	3.12	2.15	3.00	0.92	3.71	3.28	0.66	2.67	2.40	3.33	3.22	3.29	3.33
0.5	3.64	4.03	4.04	4.84	6.00	2.73	5.45	5.05	2.26	6.00	1.77	6.34	5.19	2.34	5.33	3.64	6.67	6.26	4.62	6.67
0.75	4.06	4.42	4.51	5.26	6.46	3.43	5.87	5.57	2.73	6.46	2.37	6.94	5.80	2.94	6.05	4.06	8.72	6.73	5.23	7.18

Table 10 Weighting coefficients and price components of the GenCos in case 3.

R	**n**	$A_{0,n}$	$A_{1,n}$	$A_{2,n}$	$A_{3,n}$	$A_{4,n}$	$A_{5,n}$	$A_{6,n}$
0	1	9.1797	0.1643	0.2068	0.0931	0.2770	0.0909	0.1678
	2	9.2261	0.1655	0.2083	0.0886	0.2822	0.0863	0.1692
	13	5.4624	0.0745	0.0886	0.4510	-0.1351	0.4578	0.0632
	22	11.5490	0.2216	0.2822	-0.1351	0.5397	-0.1429	0.2345
	23	5.3917	0.0727	0.0863	0.4578	-0.1429	0.4648	0.0613
	27	9.3252	0.1678	0.2114	0.0791	0.2931	0.0766	0.1719
0.25	1	10.0666	0.1408	0.1765	0.1372	0.2008	0.1365	0.1421
	2	10.1255	0.1412	0.1771	0.1346	0.2033	0.1338	0.1425
	13	5.3547	0.1098	0.1346	0.3476	0.0031	0.3516	0.1032
	22	13.0698	0.1607	0.2033	0.0031	0.3268	-0.0006	0.1668
	23	5.2651	0.1092	0.1338	0.3516	-0.0006	0.3556	0.1024
	27	10.2510	0.1421	0.1782	0.1290	0.2086	0.1280	0.1436
0. 5	all	9.9213	0.1271	0.1589	0.1589	0.1589	0.1589	0.1271
0.75	all	8.9027	0.1227	0.1534	0.1534	0.1534	0.1534	0.1227

Table 11 Weighting coefficients and price components of the LSEs which fully dispatched in case 3.

R	**n**	$A''_{1,n}$	$A''_{2,n}$	$A''_{3,n}$	$A''_{4,n}$	$A''_{5,n}$	$A''_{7,n}$	$A''_{8,n}$	$A''_{10,n}$	$A''_{12,n}$	$A''_{15,n}$	$A''_{16,n}$	$A''_{17,n}$	$A''_{18,n}$	$A''_{19,n}$	$A''_{20,n}$
0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.25	1	0.0353	0.0349	0.0348	0.0357	0.0358	0.0274	0.0241	0.0335	0.0776	0.0406	0.0273	0.0351	0.0354	0.0355	0.0355
	2	0.0354	0.0350	0.0349	0.0359	0.0360	0.0269	0.0233	0.0334	0.0811	0.0411	0.0268	0.0352	0.0355	0.0356	0.0356
	13	0.0269	0.0291	0.0295	0.0246	0.0240	0.0695	0.0878	0.0369	-0.2023	-0.0016	0.0703	0.0280	0.0266	0.0258	0.0258
	22	0.0407	0.0386	0.0382	0.0428	0.0434	0.0006	-0.0165	0.0312	0.2561	0.0674	-0.0001	0.0397	0.0409	0.0417	0.0417
	23	0.0268	0.0290	0.0294	0.0244	0.0238	0.0703	0.0890	0.0370	-0.2076	-0.0024	0.0711	0.0278	0.0265	0.0256	0.0256
	27	0.0356	0.0351	0.0350	0.0362	0.0363	0.0258	0.0216	0.0333	0.0886	0.0422	0.0256	0.0354	0.0357	0.0359	0.0359
0.5	all	-	-	-	-	0.0318	0.0318	-	0.0318	-	0.0318	-	0.0318	-	-	-
0.75	all	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 12 Weighting coefficients and price components of the LSEs which are not fully dispatched in case 3.

R	**0**	**0.25**						**0.5**	**0.75**
Bus num.	**all**	**1**	**2**	**13**	**22**	**23**	**27**	**all**	**all**
$A'''_{1,n}$	-	-	-	-	-	-	-	0.0064	0.0061
$A'''_{2,n}$	-	-	-	-	-	-	-	0.0058	0.0056
$A'''_{3,n}$	-	-	-	-	-	-	-	0.0071	0.0068
$A'''_{4,n}$	-	-	-	-	-	-	-	0.0064	0.0061
$A'''_{5,n}$	-	-	-	-	-	-	-		0.0061
$A'''_{6,n}$	-	0.0139	0.0142	-0.0028	0.0246	-0.0031	0.0146	0.0106	0.0102
$A'''_{7,n}$	-	-	-	-	-	-	-	-	0.0056
$A'''_{8,n}$	-	-	-	-	-	-	-	0.0079	0.0077
$A'''_{9,n}$	-	0.0048	0.0045	0.0227	-0.0066	0.0230	0.0041	0.0071	0.0068
$A'''_{10,n}$	-	-	-	-	-	-	-	-	0.0061
$A'''_{11,n}$	-	0.0112	0.0113	0.0015	0.0174	0.0014	0.0116	0.0091	0.0088
$A'''_{12,n}$	-	-	-	-	-	-	-	0.0091	0.0088
$A'''_{13,n}$	-	0.0189	0.0197	-0.0402	0.0566	-0.0413	0.0212	0.0091	0.0088
$A'''_{14,n}$	-	0.0172	0.0178	-00309	0.0479	-0.0318	0.0191	0.0091	0.0088
$A'''_{15,n}$	-	-	-	-	-	-	-	-	0.0051
$A'''_{16,n}$	-	-	-	-	-	-	-	0.0064	0.0061
$A'''_{17,n}$	-	-	-	-	-	-	-	-	0.0051
$A'''_{18,n}$	-	-	-	-	-	-	-	0.0071	0.0068
$A'''_{19,n}$	-	-	-	-	-	-	-	0.0091	0.0088
$A'''_{20,n}$	-	-	-	-	-	-	-	-	0.0068

6 Conclusion

This paper presented a new analytical approach along with an agent-based approach for assessing impacts of price-sensitive loads on the LMP and market power by decomposing LMP to constitutive components at market equilibrium. The proposed decomposition of the LMP indicates the impact of the bidding strategies and maximum price-sensitive demand of LSEs on the electricity price at different buses. It was demonstrated in this paper that in the presence of price-sensitive loads the LMP is composed of five constitutive components. The first component is constant while, the second component is the weighted summation of strategies of the marginal generating units and the third component is the weighted sum of power generated by the units which are bounded by their generation caps. The fourth component is the weighted sum of LSEs' maximum price-sensitive demand of fully dispatched LSEs and last component is weighted aggregation of strategies of LSEs, which are not completely dispatched. The weighting coefficients of each LSE and the price components of the LSEs at each bus can be employed by the market operator for efficient evaluation of influences of LSEs on mitigation of the market power of GenCos. The proposed decomposition and evaluation approach was applied and tested on two test system. The simulation results illustrated the efficiency of the proposed approach.

If you are using Word, use either the Microsoft Equation Editor or the MathType for equations in your paper.

Appendix A:

The Karush-Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial Q_{Si}} = 0 \Rightarrow\ a_i + b_i Q_{Si} - \lambda + \mu_i + \sum_{l=1}^{L} \Gamma_l^{max} \gamma_{l,i} - \sum_{l=1}^{L} \Gamma_l^{min} \gamma_{l,i} =$$

$$\frac{\partial L}{\partial Q_{Dj}^{s}} = 0 \Rightarrow -c_j + d_j Q_{Dj}^{S} + \lambda + \omega_j - \sum_{l=1}^{L} \Gamma_l^{max} \gamma_{l,j} + \sum_{l=1}^{L} \Gamma_l^{min} \gamma_{l,j} =$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \sum_{j\in N_D} Q_{Dj}^{S} + \sum_{j\in N_D} Q_{Dj}^{F} - \sum_{i\in N_S} Q_{Si} = 0$$

$$\mu_i \left(Q_{Si} - \overline{Q}_{Si} \right) = 0 \Rightarrow \begin{cases} \mu_i = 0 & i \in N_S - \overline{N}_S \\ \mu_i > 0 & i \in \overline{N}_S \end{cases} \quad \text{(A1)}$$

$$\omega_j \left(Q_{Dj}^{S} - Q_{Dj}^{S,max} \right) = 0 \Rightarrow \begin{cases} \omega_j = 0 & j \in N_D - \overline{N}_D \\ \omega_j > 0 & j \in \overline{N}_D \end{cases}$$

$$\Gamma_l^{max} \left(\sum_{n\in N} \gamma_{l,n} \left(Q_S^n - Q_D^n \right) - \overline{\alpha}_l \right) = 0 \Rightarrow \begin{cases} \Gamma_l^{max} = 0 & l \in L - L \\ \Gamma_l^{max} > 0 & l \in L_{cong} \end{cases}$$

$$\Gamma_l^{min} \left(\underline{\alpha}_l - \sum_{n\in N} \gamma_{l,n} \left(Q_S^n - Q_D^n \right) \right) = 0 \Rightarrow \begin{cases} \Gamma_l^{max} = 0 & l \in L - L \\ \Gamma_l^{max} > 0 & l \in L_{cong} \end{cases}$$

KKT condition for the binding inequality constraints of transmission lines:

$$\sum_{n\in N}\gamma_{l,n}\left(Q_S^n - Q_D^n\right)=\bar{\alpha}_l \qquad l\in L_{cong} \tag{A2}$$

By substituting Q_{Si} and Q_{Dj}^S from (8) and (9) into (A2), we have

$$\sum_{m\in N_S-\bar{N}_S}\gamma_{l,m}\left(\frac{\lambda - a_m - \sum_{k\in L_{cong}}\Gamma_k^{max}\gamma_{k,m}}{b_m}\right) - \sum_{r\in N_D-\bar{N}_D}\gamma_{l,r}\left(\frac{-\lambda + c_r + \sum_{k\in L_{cong}}\Gamma_k^{max}\gamma_{k,r}}{d_r}\right) = \bar{\alpha}_l + \sum_{j\in N_D}\gamma_{l,j}Q_{Dj}^F + \sum_{j\in N_D}\gamma_{l,j}Q_{Dj}^{S,max} - \sum_{i\in N_S}\gamma_{l,i}\bar{Q}_{Si} \qquad l\in L_{cong} \tag{A3}$$

After replacing λ from (12) into (A3) and some manipulation, this equation can be rewritten as:

$$\sum_{m\in N_S-\bar{N}_S}\left(\frac{\sum_{i\in N_S-\bar{N}_S}\frac{\gamma_{l,i}}{b_i}}{C_l b_m} - \frac{\gamma_{l,m}}{b_m} + \frac{\sum_{r\in N_D-\bar{N}_D}\frac{\gamma_{l,r}}{d_r}}{C_l b_m}\right)a_m + \sum_{k\in L_{cong}}\left[\sum_{m\in N_S-\bar{N}_S}\left(\frac{\sum_{i\in N_S-\bar{N}_S}\frac{\gamma_{l,m}\gamma_{k,i}}{b_i}}{C_l b_m} - \frac{\gamma_{l,m}\gamma_{k,m}}{b_m} + \frac{\sum_{j\in N_D-\bar{N}_D}\frac{\gamma_{l,m}\gamma_{k,j}}{d_j}}{C_l b_m}\right)\right]\Gamma_k^{max} + \sum_{k\in L_{cong}}\left[\sum_{r\in N_D-\bar{N}_D}\left(\frac{\sum_{i\in N_S-\bar{N}_S}\frac{\gamma_{l,r}\gamma_{k,i}}{b_i}}{C_l d_r} - \frac{\gamma_{l,r}\gamma_{k,r}}{d_r} + \frac{\sum_{j\in N_D-\bar{N}_D}\frac{\gamma_{l,r}\gamma_{k,j}}{d_j}}{C_l d_r}\right)\right]\Gamma_k^{max} = \bar{\alpha}_l + \sum_{j\in N_D}\gamma_{l,j}Q_{Dj}^F + \sum_{j\in N_D}\gamma_{l,j}Q_{Dj}^{S,max} - \sum_{i\in N_S}\gamma_{l,i}\bar{Q}_{Si} + \sum_{r\in N_D-\bar{N}_D}\gamma_{l,r}\frac{c_r}{d_r} - \left(\sum_{m\in N_S-\bar{N}_S}\frac{\gamma_{l,m}}{b_m} + \sum_{r\in N_D-\bar{N}_D}\frac{\gamma_{l,r}}{d_r}\right)D_l \tag{A4}$$

Matrices α, β, C, D, E and F from (13) are defined as:

$$\alpha(l,m) = \frac{\sum_{i\in N_S-\bar{N}_S}\frac{\gamma_{l,i}}{b_i}}{C_l b_m} - \frac{\gamma_{l,m}}{b_m} + \frac{\sum_{r\in N_D-\bar{N}_D}\frac{\gamma_{l,r}}{d_r}}{C_l b_m}$$

$$\beta(l,k) = \sum_{m\in N_S-\bar{N}_S}\left(\frac{\sum_{i\in N_S-\bar{N}_S}\frac{\gamma_{l,m}\gamma_{k,i}}{b_i}}{C_l b_m} - \frac{\gamma_{l,m}\gamma_{k,m}}{b_m} + \frac{\sum_{j\in N_D-\bar{N}_D}\frac{\gamma_{l,m}\gamma_{k,j}}{d_j}}{C_l b_m}\right) + \sum_{r\in N_D-\bar{N}_D}\left(\frac{\sum_{i\in N_S-\bar{N}_S}\frac{\gamma_{l,r}\gamma_{k,i}}{b_i}}{C_l d_r} - \frac{\gamma_{l,r}\gamma_{k,r}}{d_r} + \frac{\sum_{j\in N_D-\bar{N}_D}\frac{\gamma_{l,r}\gamma_{k,j}}{d_j}}{C_l d_r}\right)$$

$$C(l) = \bar{\alpha}_l + \sum_{j\in N_D}\left(\gamma_{l,j} - \left(\sum_{m\in N_S-\bar{N}_S}\frac{\gamma_{l,m}}{b_m} + \sum_{r\in N_D-\bar{N}_D}\frac{\gamma_{l,r}}{d_r}\right)\left(\frac{1}{C_l}\right)\right)Q_{Dj}^F$$

$$D(l,j) = \gamma_{l,j} - \frac{\sum_{m\in N_S-\bar{N}_S}\frac{\gamma_{l,m}}{b_m} + \sum_{r\in N_D-\bar{N}_D}\frac{\gamma_{l,r}}{d_r}}{C_l}$$

$$E(l,u) = \gamma_{l,u} - \frac{\sum_{m\in N_S-\bar{N}_S}\frac{\gamma_{l,m}}{b_m} + \sum_{r\in N_D-\bar{N}_D}\frac{\gamma_{l,r}}{d_r}}{C_l} \tag{A5}$$

$$F(l,z) = \frac{\gamma_{l,z}}{d_z} - \frac{\sum_{m\in N_S-\bar{N}_S}\frac{\gamma_{l,m}}{b_m} + \sum_{r\in N_D-\bar{N}_D}\frac{\gamma_{l,r}}{d_r}}{C_l d_z}$$

Matrices C_2, C_3, C_4, G, A and B_n from (16) are specified as:

$$C_2 = \frac{\sum_{j\in N_D}Q_{Dj}^F}{C_l}$$

$$C_3 = \frac{-ones\left(1, \bar{N}_S\right)}{C_l}$$

$$C_4 = \frac{ones\left(1, \bar{N}_D\right)}{C_l}$$

$$G = [G_j] = \left[\frac{1}{C_l \times d_j}\right] \tag{A6}$$

$$A = [A_i] = \left[\frac{1}{C_l \times b_i}\right]$$

$$B_n = [B_{n,l}] = \left[\frac{\sum_{i\in N_S-\bar{N}_S}\left(\frac{\gamma_{l,i}}{b_i}\right) + \sum_{j\in N_D-\bar{N}_D}\left(\frac{\gamma_{l,j}}{d_j}\right)}{C_l} - \gamma_{l,n}\right]$$

Appendix B:

Table B.1. Cost and capacity information of generators load data for IEEE-30 test system.

Bus no.	Generator			LSE	
	α ($/MWh)	b ($/MW²h)	Size (MW)	c ($/MWh)	d ($/MW²h)
1	18	0.25	100	-	-
2	20	0.2	80	48	5
3	-	-	-	52	5.5
4	-	-	-	48	4.5
7	-	-	-	54	5
8	-	-	-	60	5
10	-	-	-	38	3
12	-	-	-	60	5.5
13	25	0.2	50	-	-
14	-	-	-	50	4

15	-	-	-	40	4.5
16	-	-	-	60	5
17	-	-	-	36	3.5
18	-	-	-	52	3.5
19	-	-	-	48	3.5
20	-	-	-	38	3.5
21	-	-	-	64	6
22	22	0.2	80	-	-
23	22	0.2	50	48	5
24	-	-	-	80	6
26	-	-	-	58	4.5
27	16	0.25	120	-	-
29	-	-	-	46	3.5
30	-	-	-	60	4.5

Nomenclature

N	Set of all nodes.
N_S , N_D	Set of all generators and set of all LSEs.
$\overline{N}_S$	Set of all bounded generators.
$\overline{N}_D$	Set of all fully dispatched LSEs.
L	Set of all transmission lines.
L_{cong}	Set of all congested transmission lines.
Q_{Si} , $\overline{Q}_{Si}$	Power generated by unit i and its upper limit.
Q_S^n	Sum of generated power by GenCos at node n.
Q_{Dj}^S , $Q_{Dj}^{S,max}$	Price-sensitive load of LSE j and its maximum.
Q_D^n	Sum of demanded power by LSEs at node n.
$\underline{\alpha}_l$, $\overline{\alpha}_l$	Lower and upper limits of the flow of line l.
$\gamma_{l,n}$	Power transmission distribution factor of line l due to node n.
λ	Lagrange multiplier of the equality constraint.
μ_i , ω_j	Lagrange multiplier of inequality constraints of ISO's optimization problem.
Γ_l^{min} , Γ_l^{max}	Lagrange multipliers of the lower and upper constraints on the flow of line l.
Γ^{max}	The vector of Γ_l^{max} .

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