Robust Adaptive Actuator Failure Compensation of MIMO Systems with Unknown State Delays

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Abstract: In this paper, a robust adaptive actuator failure compensation control scheme is proposed for a class of multi input multi output linear systems with unknown time-varying state delay and in the presence of unknown actuator failures and external disturbance. The adaptive controller structure is designed based on the SPR-Lyapunov approach to achieve the control objective under the specific assumptions and the SDU factorization method of the high frequency gain matrix is employed to drive the suitable form of the error equation. The two component controller structure with an integral term is used in order to compensate the effect of unknown state delay and external disturbance. Using a suitable Lyapunov-Krasovskii functional, it is shown that despite existing external disturbance and actuator failures, all closed loop signals are bounded and the plant Output asymptotically tracks the output of a stable reference model. Simulation results are provided to demonstrate the effectiveness of the proposed theoretical results.

Keywords: Multivariable State Delay Systems, Adaptive Control, Actuator Failure Compensation.

1 Introduction

COMPONENT failures occur in many practical systems and may cause performance deterioration and even lead to system instability and catastrophic accidents. There have been many studies in the literature on control of systems with component failures [1-5]. In these papers, different design methods including multiple model, switching and tuning designs, fault detection and diagnosis designs, robust control designs and adaptive designs are used. In many applications, failures are uncertain, that is, during system operation, it is not known when components may fail, which components have failed and the extent of failures are also unknown. Adaptive control is a useful design method to handle uncertainties in both system dynamics and component failures. Some important results in the area of adaptive fault tolerant control systems exist in [6-12].

Since delay phenomena are frequently encountered in mechanics, physics, applied mathematics, biology, economics and engineering systems and time delay is a source of instability and poor performance, considerable attention has been devoted to the study of different issues related to time-delay systems [13,14]. One of these issues is the fault tolerant control of time delay systems. In the presence of time delay, the design of adaptive fault tolerant controller becomes more complex. Therefore, there are little results in this field compared with systems without delay. For example, in [15] a fault detection and accommodation method is considered for nonlinear state delay systems, based on an iterative design of an observer. The control signal is formed by treating component failures as bounded uncertainties. In [16] and [17], state feedback controllers are developed within the framework of Linear Matrix Inequalities for a class of linear systems with time delay in control inputs and constant actuator failures of stuck-type. A direct state feedback adaptive control scheme is introduced in [18] for linear state delay systems with unknown constant stuck failures in actuators. The same problem is solved for decentralized systems in [19]. Based on a linear matrix inequality technique, [20] and [21] suggest adaptive reliable controllers against loss of effectiveness actuator failures which are unknown. In this paper, the plant model is assumed to be known. In [22], an adaptive controller is designed for single input-single output (SISO) state delay systems with unknown parameters and actuator failures. An adaptive controller is designed for multi-input-multi-output (MIMO) state delay systems in [23] for known state delay and in [24] for unknown time varying state delay.

In this paper, a robust adaptive actuator failure compensation controller is designed for a certain type of multi-input multi-output (MIMO) linear systems with unknown time varying state delay. The system is considered to have $\boldsymbol{M}$ groups of inputs and $\boldsymbol{M}$ outputs. Actuators may fail in each input group, during the

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operation of the system, but at least one actuator does not fail in each group and can be used for the failure compensation. The main contribution of this paper is that considers the adaptive actuator failure compensation problem for MIMO linear time delay systems with unknown state delays and in the presence of external disturbance.

2 Problem formulation

In this section, the control problem is formulated, including the plant and reference model, actuator failure model, assumptions and control objective. Consider a linear MIMO state delay plant described by

$$\begin{aligned} &\dot{x}(t) = Ax(t) + A_d x(t - d(t)) + Bu(t) + B_f f(t), \quad t \geq 0 \\ &y(t) = Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, $y(t) \in \Re^M$ is the output vector and $u(t) \in \Re^N$ is the input vector whose elements may fail during system operation. $f(t) \in \Re^M$ is the external disturbance with $\|f(t)\| \leq f^*$. The constant matrices $A \in \Re^{n\times n}$, $A_d \in \Re^{n\times n}$, $B \in \Re^{n\times N}$, $B_f \in \Re^{n\times 1}$ and $C \in \Re^{M\times n}$ are unknown. The unknown time-varying delay $d(t)$ is a differentiable function satisfying

$$0 \leq d(t) \leq d_{\max}, \qquad \dot{d}(t) \leq \bar{d} < 1, \tag{2}$$

where $d_{\max}$ and $\bar{d}$ are some unknown positive constants.

For MIMO plant, it is considered that the N inputs can be separated into M groups. Each input group contains n_i inputs, with

$$\sum_{i=1,\dots,M} n_i = N, \quad n_i > 1, \quad i = 1,\dots,M$$

In other words, the input vector $u(t)$ can be expressed as

$$\begin{aligned} u(t) = [&u_{11}(t),\dots,u_{1n_1}(t), u_{21}(t),\dots, \\ &u_{2n_2}(t),\dots,u_{M1}(t),\dots,u_{Mn_M}(t)]^T \end{aligned} \tag{3}$$

and the constant matrix B is written as $B = [b_{11},\dots, b_{1n_1}, b_{21},\dots,b_{2n_2},\dots,b_{M1},\dots,b_{Mn_M}]$.

When there is no delay term and no disturbance, the transfer matrix of the plant (1) is described by

$$y(t) = W_o(s)u(t) \tag{4}$$

where $W_o(s) = [W_{11}(s),\dots,W_{1n_1}(s), W_{21}(s),\dots,W_{2n_2}(s), \dots,W_{M1}(s),\dots,W_{Mn_M}(s)]$ is an $M \times N$ transfer matrix.

In this paper, one important type of actuator failure modeled as

$$u_{ij}(t) = \bar{u}_{ij}, \quad t \geq t_{ij} \quad i \in \{1,2,\dots,M\}, \; j \in \{1,2,\dots,n_i\} \tag{5}$$

is considered, where the constant values $\bar{u}_{ij}$ and the failure time instants t_{ij} are unknown. Each of system actuators may fail during system operation, but at least one actuator in each group continues its reasonable operation. With this type of actuator failure, the input vector $u_i(t) = [u_{i1},\dots,u_{in_i}]^T$ is defined as

$$u_i(t) = v_i(t) + \sigma_i(\bar{u}_i - v_i(t)) \tag{6}$$

where $\bar{u}_i = [\bar{u}_{i1},\dots,\bar{u}_{in_i}]^T$, $\sigma_i = \mathrm{diag}\{\sigma_{i1},\sigma_{i2},\dots,\sigma_{in_i}\}$, $\sigma_{ij} = \begin{cases} 1, & u_{ij}(t) = \bar{u}_{ij} \\ 0, & u_{ij}(t) \neq \bar{u}_{ij} \end{cases}$ and $v_i(t) = [v_{i1},\dots,v_{in_i}]^T$ is an applied control input to be designed for group i. For this type of actuator failure, it is a basic assumption that [6]:

(A1)- If the system parameters and actuator failures (up to $n_i - 1$ failures in each group) are known, the remaining actuators can still achieve a desired control objective.

The control objective is to determine an output feedback $v(t) = [v_1^T(t), v_2^T(t),\dots,v_M^T(t)]^T$ for the plant (1) with unknown parameters and unknown actuator failures (5) such that despite the control errors $u_i - v_i = \sigma_i(\bar{u}_i - v_i)$, all signals of the closed-loop system remain bounded and the plant output vector $y(t)$ follows the output vector $y_m(t)$ of a stable reference model with the transfer matrix

$$y_m(t) = W_m(s)r(t), \tag{7}$$

asymptotically; i.e., $\lim_{t\to\infty} e(t) = \lim_{t\to\infty}(y(t) - y_m(t)) = 0$. In the above equation, $r(t)$ is the reference input which is assumed to be uniformly bounded and piecewise continuous.

In order to design the controller structure, the "equal control" design

$$v_{i1}(t) = v_{i2}(t) = \dots = v_{in_i}(t) \equiv v_{io}(t), \quad i = 1,\dots,M \tag{8}$$

is chosen which assumes that the control inputs to all actuators of each group are the same. It is reasonable in many practical applications. For example, segments of a multiple-segment rudder or heating devices of an oven have similar physical characteristics. With this actuation scheme, when there is no actuator failure, the transfer function of the system without delay (4) can be stated as

$$y(t) = W(s)v_o(t) \tag{8}$$

where $v_o(t) = [v_{1o}(t), v_{2o}(t), ..., v_{Mo}(t)]^T$ and

$$W(s) = [\sum_{j=1,...,n_1} W_{1j}(s), ..., \sum_{j=1,...,n_M} W_{Mj}(s)] \tag{9}$$

is an $M \times M$ transfer matrix.

With the assumption that at time instant t, p_i actuators fail in each group and there are totally $p = \sum_{i=1,...,M} p_i$ failed actuators, i.e. $u_{ij} = \bar{u}_{ij}$, $i = 1,...,M$, $j = j_{i1},..., j_{ip_i}$, $0 \le p_i \le n_i - 1$.

Then from (6) and (8) the closed-loop system (9) can be expressed as

$$y(t) = W_a(s)v_o(t) + \bar{y}(t) \tag{10}$$

where $W_a(s) = [\sum_{j \ne j_{11},...,j_{1p_1}} W_{1j}(s), ..., \sum_{j \ne j_{M1},...,j_{Mp_M}} W_{Mj}(s)]$ is an $M \times M$ transfer matrix with the state space representation $C(sI - A)^{-1}B_a$, $B_a = [\sum_{j \ne j_{11},...,j_{1p_1}} b_{1j}, ..., \sum_{j \ne j_{M1},...,j_{Mp_M}} b_{Mj}]$ and

$$\bar{y}(t) = \sum_{i=1,...,M} \sum_{j=j_{i1},...,j_{ip_i}} G_{ij}(s)\bar{u}_{ij} \tag{11}$$

To design the controller structure to meet the control objective, it is assumed that $W_a(s)$ satisfies the following assumptions for each failure pattern:

(A2)- The transmission zeros of $W_a(s)$ have negative real parts.

(A3)- An upper bound $\bar{v}_0$ on the observability indices of all possible $W_a(s)$ is known.

(A4)- $W_a(s)$ is strictly proper, has full rank and has relative degree 1 for each failure pattern.

(A5)- Because of the assumption (A4) and without loss of generality, the referenced model is selected as

$$W_m(s) = \text{diag}[\frac{1}{s + a_{mi}}], \quad a_{mi} > 0, \quad i = 1,...,M \tag{12}$$

(A6)- All leading principal minors of the high-frequency gain matrix $K_{pa} = \lim_{s \to \infty} sW_a(s)$ are nonzero and their signs are known and do not change as actuator failure patterns change.

(A7)- There exist column proper $M \times M$ polynomial matrix $D_r(s)$ and row proper $M \times M$ polynomial matrix $D_l(s)$ for all failure patterns such that

$$W_a(s) = N_{ra}(s)D_r^{-1}(s) = D_l^{-1}(s)N_{la}(s) \tag{13}$$

where $N_{ra}(s)$ and $N_{la}(s)$ are $M \times M$ polynomial matrices associated with each failure pattern. $N_{ra}(s)$ and $D_r(s)$ are right coprime and $N_{la}(s)$ and $D_l(s)$ are left coprime of matrix $W_a(s)$.

3 Plant-model matching control

In this section, we consider the system without delay (9) and when the plant parameters and actuator failures are known. But the results of this section are only used to obtain a suitable error equation parameterization and design the adaptive controller for system with delayed states and unknown parameters and actuator failures in the next sections. Therefore, the final controller structure doesn't need the plant parameters information to implement. The control input is denoted as

$$v_o(t) = v_o^*(t) = [v_{1o}^*(t), v_{2o}^*(t), ..., v_{Mo}^*(t)]^T$$

and the controller structure is defined as

$$\begin{aligned} & v_o^*(t) = K_e^* y(t) + K_1^{*T} x_1(t) + K_2^{*T} x_2(t) + K_r^* r(t) + K_3^*, \\ & x_1(t) = H_M(s)v_o(t), \\ & x_2(t) = H_M(s)y(t), \\ & H_M(s) = \frac{\alpha(s)}{\Lambda(s)}, \end{aligned} \tag{14}$$

in which $\alpha(s) = [I_{M \times M}, sI_{M \times M}, ... s^{\bar{v}_0 - 2} I_{M \times M}]^T$ and $\Lambda(s)$ is a monic Hurwitz polynomial of degree $\bar{v}_0 - 1$, $K_1^* = [K_{11}^{*T}, K_{12}^{*T}, ..., K_{1(\bar{v}_0-1)}^{*T}] \in \Re^{M(\bar{v}_0 - 1) \times M}$, $K_2^* = [K_{21}^{*T}, K_{22}^{*T}, ..., K_{2(\bar{v}_0-1)}^{*T}] \in \Re^{M(\bar{v}_0 - 1) \times M}$, $K_e^* \in \Re^{M \times M}$ and $K_r^* \in \Re^{M \times M}$ are the parameters of the controller structure introduced in [25] for MRC of systems without delay and the additional term $K_3^* \in \Re^M$ is a constant that is chosen for the compensation of the control error $u_i - v_i = \sigma_i(\bar{u}_i - v_i)$.

In order to drive the controller parameters, $y(t)$ from (10) is substituted in (14) and the control signal $v_o^*(t)$ is obtained as

$$\begin{aligned} v_o^*(t) = & (I - K_1^{*T} H_M(s) - K_2^{*T} H_M(s)W_a(s) \\ & - K_e^* W_a(s))^{-1} \\ & \times [K_2^{*T} H_M(s)\bar{y} + K_e^* \bar{y} + K_r^* r + K_3^*]. \end{aligned} \tag{15}$$

Therefore, the closed-loop system (10) becomes

$$y(t)=W_a(s)(I-K_1^{*T}H_M(s)-K_2^{*T}H_M(s)W_a(s)-K_e^{*}W_o(s))^{-1}\times[K_2^{*T}H_M(s)\bar{y}+K_e^{*}\bar{y}+K_r^{*}r+K_3^{*}]+\bar{y}(t) \quad (16)$$

With the definition of $\Lambda(s)$, $H_M(s)$, $W_m(s)$, $D_r(s)$ and $N_a(s)$ there exist K_1^*, K_2^*, K_e^* and $K_r^*=K_{pa}^{-1}$ such that [6]

$$\begin{aligned}&K_1^{*T}\alpha(s)D_r(s)+(K_2^{*T}\alpha(s)+K_e^*\Lambda(s))N_{ra}(s)\\&=\Lambda(s)(D_r(s)-K_r^*W_m^{-1}(s)N_{ra}(s)),\end{aligned} \quad (17)$$

Therefore, using the similar discussions as that of [25], the closed-loop system equation (10) can be written as

$$y(t)=W_m(s)r(t)+f_p(t), \quad (18)$$

with

$$\begin{aligned}f_p(t)=W_m(s)K_{pa}[&K_2^{*T}H_M(s)\bar{y}+K_e^*\bar{y}\\&+K_r^*W_m^{-1}(s)\bar{y}+K_3^*](t).\end{aligned} \quad (19)$$

According to assumption (A7) and from (11), $\bar{y}$ can be describe as

$$\bar{y}(t)=D_l^{-1}(s)\sum_{i=1,\dots,M}\ \sum_{j=j_{i1},\dots,j_{ip_i}}N_{ij}(s)\bar{u}_{ij}, \quad (20)$$

and consequently, $f_p(t)$ from (19) is rewritten as

$$\begin{aligned}f_p(t)=W_m(s)K_{pa}[&\frac{\Lambda(s)I-K_1^{*T}\alpha(s)}{\Lambda(s)}N_{la}^{-1}(s)\\&\times\sum_{i=1,\dots,M}\ \sum_{j=j_{i1},\dots,j_{ipi}}N_{ij}(s)\bar{u}_{ij}+K_3^*](t).\end{aligned} \quad (21)$$

Since $W_m(s)$, $\Lambda(s)$ and $N_{la}(s)$ are all stable, it can be concluded that there exists a constant K_3^* such that $f_p(t)$ converges to zero exponentially. In fact, because $\bar{u}_{ij}$ is constant, the term $\sum_{i=1,\dots,M}\sum_{j=j_{i1},\dots,j_{ipi}}N_{ij}(s)\bar{u}_{ij}$ is a constant value also. Therefore, the output of

$$\frac{\Lambda(s)I-K_1^{*T}\alpha(s)}{\Lambda(s)}N_{la}^{-1}(s)\times\sum_{i=1,\dots,M}\ \sum_{j=j_{i1},\dots,j_{ipi}}N_{ij}(s)\bar{u}_{ij}$$

which is a stable transfer function with constant input, converges to a constant value. If the constant K_3^* be chosen to be the negative of this value, $f_p(t)$ converges to zero exponentially. Thus, we have: $\lim_{t\to\infty}(y(t)-y_m(t))=0$ and plant model matching is achieved.

Remark 1. Suppose that the actuator failures occur at the times T_i, $i=1,\dots,m_0$, with $m_0<N-M+1$ since at least one actuator in each group does not fail. Then (T_i,T_{i+1}), $i=0,\dots,m_0$ with $T_0=0$ and $T_{m_0+1}=\infty$ are the time intervals on which the actuator failure pattern is fixed. Since the actuator failure pattern changes at times T_i, $i=1,\dots,m_0$, the parameters of the transfer matrix $W_a(s)$, and hence $Z_a(s)$, K_{pa} and the controller parameters K_e^*, K_1^*, K_2^*, K_r^*, and K_3^* also change, thus, they are all piecewise constant parameters.

4 Error equation

Now consider the system with state delay (1) and when the system parameters and actuator failures are unknown and suppose that that p_i actuators fail in each group. With the assumption that there exist constant matrices a_d^* and F of appropriate dimensions such that

$$A_d=B_a a_d^{*T},\quad B_f=B_aF,$$

and from (6) and (8) the closed-loop system (1) can be expressed as

$$\begin{aligned}y(t)=&W_a(s)v_o(t)+W_a(s)a_d^{*T}x(t-d(t))\\&+W_a(s)Ff(t)+\bar{y}(t).\end{aligned} \quad (22)$$

Operating both sides of (17) on $y(t)$ and using (10) and (22) we have

$$\begin{aligned}&K_1^{*T}H_M(s)N_{ra}(s)v_0(t)+\\&K_1^{*T}H_M(s)N_{ra}(s)a_d^{*T}x(t-d(t))+\\&K_1^{*T}H_M(s)N_{ra}(s)Ff(t)+\\&K_1^{*T}H_M(s)\bar{y}(t)+K_2^{*T}H_M(s)N_{ra}(s)y(t)+\\&K_e^*N_{ra}(s)y(t)=N_{ra}(s)v_0(t)+\\&N_{ra}(s)a_d^{*T}x(t-d(t))+N_{ra}(s)Ff(t)+\\&D_r(s)\bar{y}(t)-K_r^*N_{ra}(s)W_m^{-1}(s)y(t).\end{aligned} \quad (23)$$

Because $N_{uf}(s)$ and $W_m(s)$ are stable, by dividing both sides of the equality (19) on $N_{ra}(s)W_m^{-1}(s)$, $y(t)$ is obtained as

$$\begin{aligned}y(t)=&W_m(s)K_{pa}[v_o(t)-K_1^{*T}H_M(s)v_o(t)-\\&K_2^{*T}H_M(s)y(t)-K_e^*y(t)-K_3^*+\\&a_d^{*T}x(t-d(t))-K_1^{*T}H_M(s)a_d^{*T}x(t-d(t))+\\&(I-K_1^{*T}H_M(s))Ff(t)]+\\&W_m(s)K_{pa}\times\\&\left(\frac{\left(\Lambda(s)-K_1^{*T}\alpha(s)\right)D_r(s)}{\Lambda(s)}N_{ra}^{-1}+K_3^*\right)+\varepsilon(t),\end{aligned} \quad (24)$$

in which $\varepsilon(t)$ is related to the system initial conditions. Based on the analysis of [26] and as it is stated in [6], $\varepsilon(t)$ converges to zero exponentially. By ignoring exponentially decaying terms $f_p(t)$ and $\varepsilon(t)$, the tracking error $e(t)=y(t)-y_m(t)$ equation can be written as

$$\begin{aligned} e(t) = & W_m(s)K_{pa}[v_o(t)-K_e^* y(t)-K_1^{*T}x_1(t) \\ & -K_2^{*T}x_2(t)-K_r^* r(t)-K_3^*+a_d^{*T}x(t-d(t)) \\ & -K_1^{*T}H_M(s)a_d^{*T}x(t-d(t)) \\ & +(I-K_1^{*T}H_M(s))Ff(t)]. \end{aligned} \tag{26}$$

To find a suitable error equation parameterization the dynamic system

$$z(t)=K_1^{*T}H_M(s)[a_d^{*T}x(t-d(t))]=K_z^{*T}z_x(t) \tag{27}$$

is defined in which,

$$\begin{aligned} & K_z^{*T}=[K_{11}^{*T}a_d^{*T},K_{12}^{*T}a_d^{*T},...,K_{1(\bar{v}_0-1)}^{*T}a_d^{*T}], \\ & z_x(t)=H_n(s)[x(t-d(t))], \\ & H_n(s)=\frac{[I_{n\times n}s^{\bar{v}_0-2},...,I_{n\times n}s,I_{n\times n}]}{\Lambda(s)}\in\Re^{n(\bar{v}_0-1)\times n}. \end{aligned} \tag{28}$$

By decomposing $z_x(t)$ into two components as

$$\begin{aligned} & z_x(t)=z_e(t)+z_m(t), \\ & z_m(t)=H_n(s)[x_m(t-d(t))], \\ & z_e(t)=H_n(s)[e_x(t-d(t))], \\ & e_x(t-d(t))=x(t-d(t))-x_m(t-d(t)), \end{aligned} \tag{29}$$

where $x_m(t)\in\Re^n$ is the state of the reference model (5). Using (27) and (29), the error equation (26) can be rewritten as follows

$$\begin{aligned} e(t) = & W_m(s)K_{pa}[v_o(t)-K_e^* e(t)-K_1^{*T}x_1(t)+ \\ & K_2^{*T}x_2(t)-K_r^* r(t)-K_3^*] \\ & -W_m(s)K_{pa}[K_m^{*T}w_m(t)+ \\ & K_d^{*T}e_x(t-d(t))+ \\ & K_z^{*T}z_e(t)+ \\ & (I-K_1^{*T}H_M(s))Ff(t)], \end{aligned} \tag{30}$$

where $K_d^*=-a_d^*$, $K_{x_m}^*=c_m^T K_e^{*T}$ and

$$\begin{aligned} & K_m^*=[K_{x_m}^{*T},K_d^{*T},K_z^{*T}]^T, \\ & w_m(t)=[x_m^T(t),x_m^T(t-d(t)),z_m^T(t)]^T. \end{aligned}$$

Now the SDU factorization of K_{pa} is employed to drive the suitable form of the error equation. For this purpose, the following two lemmas are needed.

Lemma 1 ([27]). Every $M\times M$ real matrix K_p with nonzero leading principle minors $\Delta_1,\Delta_2,..,\Delta_M$ can be factored as

$$K_p=SDU \tag{31}$$

where S is symmetric positive definite, U is unity upper triangular and $D=\operatorname{diag}\{d_1,...,d_M\}$ is diagonal with

$$\begin{aligned} & sign(d_l)=sign(\frac{\Delta_l}{\Delta_{l-1}}),\ l=2,...,M, \\ & sign(d_1)=sign(\Delta_1). \end{aligned}$$

Lemma 2 ([27]). For any $W_m(s)$ from (7), there exists a positive definite matrix $S=S^T$ such that $W_m(s)S$ is SPR.

From assumption (A6) and using Lemma 1, for any possible failure pattern the high frequency gain matrix K_{pa} has the SDU factorization

$$K_{pa}=S_a D_a U_a \tag{32}$$

where both symmetric positive definite matrix S_a and unit upper triangular matrix U_a can be unknown and are allowed to change with failure patterns. The sign of the entries d_{ai}, $i=1,...,M$ of the diagonal matrix $D_a=\operatorname{diag}\{d_{a1},...,d_{aM}\}$ is the only information that is needed for an adaptive control design and is determined by the sign of the leading principle minors of K_{pa}. According to assumption (A6), the sign of D_a is known and dose not change when actuator failure pattern changes.

By substituting the high frequency gain matrix decomposition (32) in (30) and introducing the decomposition $U_a v_o=v_o-(I-U_a)v_o$, the error equation

$$\begin{aligned} e(t) = & W_m(s)S_a D_a[v_o(t)-(I-U_a)v_o(t)- \\ & U_a K_e^* e(t)-U_a K_1^{*T}x_1(t)- \\ & U_a K_2^{*T}x_2(t)-U_a K_r^* r(t)- \\ & U_a K_3^*] \\ & -W_m(s)S_a D_a[U_a K_m^{*T}w_m(t)+ \\ & U_a K_d^{*T}e_x(t-d(t))+ \\ & U_a K_z^{*T}z_e(t)+ \\ & U_a(I-K_1^{*T}H_M(s))Ff(t)], \end{aligned} \tag{33}$$

is obtained. By defining $\theta_e^*=U_a K_e^*$, $\theta_1^{*T}=U_a K_1^{*T}$, $\theta_2^{*T}=U_a K_2^{*T}$, $\theta_r^*=U_a K_r^*$, $\theta_3^*=U_a K_3^*$, $\theta_u^*=(I-U_a)$,

$\theta_m^{*T} = U_a K_m^{*T}$, $\theta_d^{*T} = U_a K_d^{*T}$, $\theta_z^{*T} = U_a K_z^{*T}$ and $\mu(t) = (I - K_1^{*T} H_M(s))Ff(t)$ the above equation is rewritten as

$$\begin{aligned} e(t) = & W_m(s)S_a D_a[v_o(t) - K^{*T} w(t)] \\ & - W_m(s)S_a D_a[\theta_m^{*T} w_m(t) + \\ & \qquad \theta_d^{*T} e_x(t - d(t)) + \\ & \qquad \theta_z^{*T} z_e(t) + U_a \mu(t)], \end{aligned} \tag{34}$$

where

$$K^* = [\theta_e^*, \theta_1^{*T}, \theta_2^{*T}, \theta_r^*, \theta_3^*, \theta_u^*]^T,$$
$$w(t) = [e^T(t), x_1^T(t), x_2^T(t), r^T(t), 1, v_o^T(t)]^T.$$

Noting that θ_u^* is strictly upper triangular, as in [27], the new parameterization

$$K^{*T} w(t) = [\Theta_1^{*T} \Omega_1(t), \Theta_2^{*T} \Omega_2(t), \ldots, \Theta_M^{*T} \Omega_M(t)]^T, \tag{35}$$

is introduced in order to remove the zero entries from θ_u^*. Each row vector Θ_i^{*T} is obtained by concatenating the i th row of the matrices θ_e^*, θ_1^{*T}, θ_2^{*T}, θ_r^* and θ_3^* together with the nonzero entries of the i th row of θ_u^*. The corresponding regressor vectors are

$$\begin{aligned} &\Omega_1(t) = [e^T(t), x_1^T(t), x_2^T(t), r^T(t), 1, v_{o2}(t), v_{o3}(t), \ldots, v_{oM}(t)]^T, \\ &\Omega_2(t) = [e^T(t), x_1^T(t), x_2^T(t), r^T(t), 1, v_{o3}(t), \ldots, v_{oM}(t)]^T, \\ &\vdots \\ &\Omega_M(t) = [e^T(t), x_1^T(t), x_2^T(t), r^T(t), 1]^T. \end{aligned} \tag{36}$$

5 Adaptive controller

In this section, adaptive controller is designed for system with delayed states and unknown parameters and actuator failures. In view of the parameterization (35), the controller structure

$$\begin{aligned} v_o(t) = & [\Theta_1^T \Omega_1(t), \Theta_2^T \Omega_2(t), \ldots, \Theta_M^T \Omega_M(t)]^T \\ & - K_I sign(e(t)) \int_0^t |e(\tau)| d\tau, \end{aligned} \tag{37}$$

is suggested where $\Theta_i(t)$ is the estimate of Θ_i^* and K_I is a diagonal matrix with constant entries k_{Ik}, $k = 1, \ldots, M$. $\mathrm{sign}(e(t))$ is a diagonal matrix of the form

$$\mathrm{sign}(e(t)) = \mathrm{diag}[\mathrm{sign}(e_1(t), \ldots, \mathrm{sign}(e_M(t))]$$

with

$$\mathrm{sign}(e_i(t)) = \begin{cases} 1 & e_i(t) > 0 \\ 0 & e_i(t) = 0 \\ -1 & e_i(t) < 0 \end{cases}$$

and $\int_0^t |e(\tau)| d\tau = \left[\int_0^t |e_1(\tau)| d\tau, \ldots, \int_0^t |e_M(\tau)| d\tau \right]^T$.

The control law (37) is composed of two terms. The first component is the same as the controller structure introduced in [20] for actuator failure compensation of MIMO systems with the difference that the output vector $y(t)$ is replaced with the error vector $e(t)$ in the regressor vector $w(t)$. The integral term $K_I \mathrm{sign}(e(t)) \int_0^t |e(\tau)| d\tau$ is used to achieve robustness with respect to unknown plant delay and external disturbance.

Introducing the parameter errors $\tilde{\Theta}_i(t) = \Theta_i(t) - \Theta_i^*$ and using (35) and (37), the tracking error (36) is rewritten as

$$\begin{aligned} e(t) = & W_m(s)S_a D_a[(\tilde{\Theta}_1^T \Omega_1(t), \tilde{\Theta}_2^T \Omega_2(t), \ldots, \tilde{\Theta}_M^T \Omega_M(t))^T \\ & - K_I sign(e(t)) \int_0^t |e(\tau)| d\tau] \\ & - W_m(s)S_a D_a[\theta_m^{*T} w_m(t) + \theta_d^{*T} e_x(t - d(t)) \\ & + \theta_z^{*T} z_e(t) + U_a \mu(t)], \end{aligned} \tag{38}$$

Now the augmented state vector $\hat{x}(t) = [x^T(t), x_1^T(t), x_2^T(t)]^T$ is defined. Let $\hat{e}(t) = \hat{x}(t) - \hat{x}_m(t)$ where $\hat{x}_m(t)$ is the state of a nonminimal realization $\hat{C}(sI - \hat{A})^{-1}\hat{B}$ of $W_m(s)S_a$. Then the state space representation

$$\begin{aligned} \dot{\hat{e}}(t) = & \hat{A}\hat{e}(t) + \hat{B}D_a[(\tilde{\Theta}_1^T \Omega_1(t), \tilde{\Theta}_2^T \Omega_2(t), \ldots, \tilde{\Theta}_M^T \Omega_M(t))^T \\ & - K_I sign(e(t)) \int_0^t |e(\tau)| d\tau] \\ & - \hat{B}D_a[\theta_m^{*T} w_m(t) + \theta_d^{*T} e_x(t - d(t)) \\ & + \theta_z^{*T} z_e(t) + U_a \mu(t)], \\ \dot{\hat{z}}_e(t) = & A_e \hat{z}_e(t) + B_e L^T \hat{e}(t - d(t)), \\ z_e(t) = & C_e \hat{z}_e(t), \\ e(t) = & \hat{C}\hat{e}(t), \end{aligned} \tag{39}$$

is obtained for (28), where $L = [I_{n \times n}, 0_{n \times M(\bar{v}_0 - 1)}, 0_{n \times M(\bar{v}_0 - 1)}]^T$ and the triple (A_e, B_e, C_e) is a minimal state space realization for the stable transfer matrix $H_n(s)$.

Because $W_m(s)S_a = \hat{C}(sI - \hat{A})^{-1}\hat{B}$ is SPR [28], there exist matrices $P = P^T > 0$, and $\hat{Q} = \hat{Q}^T > 0$ satisfying

$$\begin{aligned} &\hat{A}^T P + P\hat{A} = -\hat{Q}, \\ &P\hat{B} = \hat{C}^T. \end{aligned} \tag{40}$$

For the next discussions, $\hat{Q}$ is considered to be the sum of two positive symmetric matrices Q and $\bar{Q}$

$$\hat{Q} = Q + \bar{Q}, \quad Q = Q^T > 0, \quad \bar{Q} = \bar{Q}^T > 0 \tag{41}$$

Since A_e is stable, there exist symmetric positive definite matrices $P_z = P_z^T > 0$ and $Q_z = Q_z^T > 0$ that satisfy

$$A_e^T P_z + P_z A_e = -Q_z . \tag{42}$$

Now we are ready to state the following theorem.

Theorem 1: Consider the system (1) with actuator failures (5) and the reference model (7). Suppose that assumptions (A1) to (A7) hold. Then for positive constants γ_i and γ_{Ii}, $i = 1,...,M$ the adaptive control (37) with coefficients

$$\begin{aligned} \dot{\Theta}_i(t) &= -\gamma_i \operatorname{sign}(d_{ai})\Omega_i(t)e_i(t) \\ k_{Ii} &= \gamma_{Ii} \operatorname{sign}(d_{ai}), \end{aligned} \tag{43}$$

assures that all the closed-loop signals are bounded and the tracking error *e(t)* converges to zero asymptotically.

Proof To prove this theorem, the Lyapunov-Krasovskii functional

$$\begin{aligned} V(t) &= \hat{e}^T(t)P\hat{e}(t) + \hat{z}_e^T(t)P_z\hat{z}_e(t) \\ &+ \int_{t-d(t)}^{t} \hat{e}^T(s)\bar{Q}\hat{e}(s)ds \\ &+ \sum_{i=1}^{M} \gamma_i^{-1}|d_{ai}|(\tilde{\Theta}_i - \bar{K}_i)^T(\tilde{\Theta}_i - \bar{K}_i) \\ &+ \sum_{i=1}^{M} \gamma_{Ii}^{-1}|d_{ai}|(-\gamma_{Ii}\int_0^t |e_i(t)|dt + \eta^*)^2 \end{aligned} \tag{44}$$

is chosen in which γ_i and γ_{Ii}, $i = 1,...,M$ are positive constant scalars and the parameter $\eta^* > 0$ with arbitrary value will be defined later. The vectors $\bar{K}_i$ are defined as

$$\bar{K}_i = -\frac{r}{2} d_{ai}^{-1}[I_{M \times M}, 0,...,0]^T$$

where $r > 0$ is an as yet unspecified constant scalar.

With this definition we have

$$\begin{aligned} -2\sum_{i=1}^{M} \gamma_i^{-1}|d_{ai}|\bar{K}_i^T\dot{\tilde{\Theta}}_i &= 2\sum_{i=1}^{M} d_{ai}\bar{K}_i\Omega_i e_i \\ &= -r\hat{e}^T(t)P\hat{B}\hat{B}^T P\hat{e}. \end{aligned} \tag{45}$$

Also since D_a is diagonal,

$$\begin{aligned} -2\sum_{i=1}^{M} d_{ai}\tilde{\Theta}_i^T\Omega_i e_i &= \\ -2\hat{e}^T(t)P\hat{B}D_a[\tilde{\Theta}_1^T\Omega_1,...,\tilde{\Theta}_M^T\Omega_M]^T&. \end{aligned} \tag{46}$$

According to the update law (43) and using (40), (42), (45) and (46), the time derivative of *V(t)* along (39) is

$$\begin{aligned} \dot{V}(t) = &-\hat{e}^T(t)Q\hat{e}(t) \\ &-(1-\dot{d}(t))\hat{e}^T(t-d(t))\bar{Q}\hat{e}(t-d(t)) \\ &-r\hat{e}^T(t)P\hat{B}\hat{B}^T P\hat{e}(t) - \hat{z}_e^T(t)Q_z\hat{z}_e(t) \\ &-2\hat{e}^T(t)P\hat{B}D_a\theta_d^{*T}L^T\hat{e}(t-d(t)) \\ &-2\hat{e}^T(t)P\hat{B}D_a\theta_z^{*T}C_e\hat{z}_e(t) \\ &-2\hat{e}^T(t)P\hat{B}D_a\theta_m^{*T}w_m(t) \\ &-2\hat{e}^T(t)P\hat{B}D_aU_a\mu(t) \\ &+2\hat{z}_e^T(t)P_zB_eL^T\hat{e}(t-d(t)) \\ &-2\hat{e}^T(t)P\hat{B}D_a[k_{I1}\operatorname{sign}(e_1(t))\int_0^t|e_1(t)|dt,... \\ &\qquad ,k_{IM}\operatorname{sign}(e_M(t))\int_0^t|e_M(t)|dt]^T \\ &-2\sum_{i=1}^{M}|d_{ai}||e_i(t)|(-\gamma_{Ii}\int_0^t|e_i(t)|dt + \eta^*) \end{aligned} \tag{47}$$

for $t \in (T_i, T_{i+1})$, $i = 0,1,...,m_0$.

Because $W_m(s)$ and $H_n(s)$ are stable and the reference input $r(t)$ is bounded, the reference signals $x_m(t)$, $x_m(t-d)$ and $z_m(t)$ are bounded. Therefore there exists a constant w_m^* such that $\|w_m(t)\| \le w_m^*$ and we can write

$$\begin{aligned} &-2\hat{e}^T(t)P\hat{B}D_a\theta_m^{*T}w_m(t) \le \\ &\quad 2\left|\hat{e}^T(t)P\hat{B}D_a\right|\left\|\theta_m^{*T}\right\|\|w_m(t)\| \le \\ &\qquad 2\left|e^T(t)D_a\right|\left\|\theta_m^{*T}\right\|w_m^* \end{aligned} \tag{48}$$

for the seventh term of (47). By choosing $\eta_1^* = \left\|\theta_m^{*T}\right\|w_m^*$, we have

$$\begin{aligned} -2\hat{e}^T(t)P\hat{B}D_a\theta_m^{*T}w_m(t) &\le 2\eta_1^*\left|e^T(t)D_a\right| \\ &= 2\sum_{i=1}^{M}|d_{ai}||e_i(t)|\eta_1^*. \end{aligned} \tag{49}$$

For the eighth term of (47), the inequality

$$\begin{aligned} &-2\hat{e}^T(t)P\hat{B}D_aU_a\mu(t) \le \\ &\quad 2\left|e^T(t)D_a\right|\left\|\left(I - K_1^{*T}H_M(s)\right)F\right\|f^* \\ &\quad = 2\eta_2^*\left|e^T(t)D_a\right| = 2\sum_{i=1}^{M}|d_{ai}||e_i(t)|\eta_2^* \end{aligned} \tag{50}$$

with $\eta_2^* = \left\|\left(I - K_1^{*T}H_M(s)\right)F\right\|f^*$ can be written.

According to the inequality

$$\pm 2x^T y \le x^T S x + y^T S^{-1} y$$

that is true for any vectors x, y and any positive definite matrix S of appropriate dimensions, the following expressions can be written for the fifth, sixth and ninth terms

$$\begin{aligned}
&-2\hat{e}^T(t)P\hat{B}D_a\theta_d^{*T}L^T\hat{e}(t-d(t)) \le \\
&\hat{e}^T(t)P\hat{B}\Psi_1\hat{B}^TP\hat{e}(t)+\hat{e}^T(t-d(t))S\hat{e}(t-d(t)), \\
&-2\hat{e}^T(t)P\hat{B}D_a\theta_z^{*T}C_e\hat{z}_e(t) \le \\
&\hat{e}^T(t)P\hat{B}\Psi_2\hat{B}^TP\hat{e}(t)+\hat{z}_e^T(t)S\hat{z}_e(t), \\
&\;2\hat{z}_e^T(t)P_zB_eL^T\hat{e}(t-d(t)) \le \\
&\hat{z}_e^T(t)\Psi_3\hat{z}_e(t)+\hat{e}^T(t-d(t))S\hat{e}(t-d(t)),
\end{aligned} \tag{51}$$

where

$$\begin{aligned}
\Psi_1 &= D_a\theta_d^{*T}L^TS^{-1}L\theta_d^*D_a^T \\
\Psi_2 &= D_a\theta_z^{*T}C_eS^{-1}C_e^T\theta_z^*D_a^T \\
\Psi_3 &= P_zB_eL^TS^{-1}LB_e^TP_z^T.
\end{aligned} \tag{52}$$

Using (49), (50) and (51), choosing the coefficients k_{Ii} from (43) and defining $\eta^* = \eta_1^* + \eta_2^*$, the inequality

$$\begin{aligned}
\dot{V}(t) \le &-\hat{e}^T(t)Q\hat{e}(t) \\
&-\hat{e}^T(t-d)(d^*\bar{Q}-2S)\hat{e}(t-d(t)) \\
&-\hat{e}^T(t)P\hat{B}(r-\Psi_1-\Psi_2)\hat{b}^TP\hat{e}(t) \\
&-\hat{z}_e^T(t)(Q_z-\Psi_3-S)\hat{z}_e(t)
\end{aligned} \tag{53}$$

is obtained, where $d^* = 1-\bar{d}$. If the arbitrary values $\bar{Q}$, r and Q_z be chosen as

$$\begin{aligned}
&\lambda_{\min}(\bar{Q}) > \lambda_{\max}(2S), \\
&r > \lambda_{\max}(\Psi_1+\Psi_2), \\
&\lambda_{\min}(Q_z) > \lambda_{\max}(\Psi_3+S)
\end{aligned}$$

we have $\dot{V}(t) \le 0$ for $t \in (T_i, T_{i+1})$, $i = 0,1,\dots,m_0$.

Since there are only a finite number of failures in system, $V(T_{m_0})$ is finite and from

$$\dot{V}(t) \le 0, \quad t \in (T_{m_0}, \infty) \tag{54}$$

we have $V(t) \in L_\infty$ and therefore $\hat{e}(t)$, $e(t)$, $\hat{z}_e(t)$, $\Theta(t)$, $\dot{\hat{\Theta}}(t) \in L_\infty$. Because $\hat{e}(t) = \hat{x}(t) - \hat{x}_m(t)$ and $\hat{x}_m(t)$ is bounded, $\hat{x}(t) = [x^T(t), x_1^T(t), x_2^T(t)]^T \in L_\infty$, which implies that $x(t), x_1(t), x_2(t)$ and $y(t) \in L_\infty$. Since $r(t)$ is uniformly bounded by assumption, $\Omega_M(t) = [e(t), x_1^T(t), x_2^T(t), r(t), 1]$ and consequently $v_{oM}(t) = [\Theta_M^T\Omega_M]$ is bounded. Therefore, $\Omega_{M-1}^T(t) = [\Omega_M^T \;\; u_M]$ is bounded. Repeating this argument, it is shown that $\Omega_M, \dots, \Omega_1$ and $v_{oM}, \dots, v_{o1}$ are all bounded. Therefore, all the signals in the closed loop system are bounded.

From (44) and (53) we establish that $\hat{e}(t)$ and therefore $e(t) \in L_2$. Using the boundedness of signals in (39) it can be concluded that $\dot{\hat{e}}(t)$ and $\dot{e}(t) \in L_\infty$. Hence, $e(t), \dot{e}(t) \in L_\infty$ and $e(t) \in L_2$, which by Barbalat's lemma [28] imply that $\lim_{t\to\infty} e(t) = 0$.

6 Simulation results

To verify the performance of the proposed adaptive controller, consider system (1) with the parameters

$$\begin{aligned}
A &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \\
A_d &= \begin{bmatrix} 0.1 & -0.4 & 0.4 \\ 0.1 & -0.7 & 0.3 \\ 0.2 & 0.3 & 0.1 \end{bmatrix}, \\
B &= [b_{11} \;\; b_{12} \;\; b_{21} \;\; b_{22}] = \begin{bmatrix} 0.5 & 1 & 0.25 & 0.75 \\ 1 & 2 & 1.5 & 4.5 \\ 1.5 & 3 & 1 & 3 \end{bmatrix} \\
B_f &= \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \\
C &= \begin{bmatrix} 1 & 1 & 0.5 \\ 1 & 0.5 & 0.25 \end{bmatrix}, \\
&d(t) = 4 + 0.5\sin(t),\; f(t) = 0.4\sin(0.1t) + 0.3, \\
&x_0 = [0.1 \;\; 0 \;\; 0.5].
\end{aligned} \tag{55}$$

With these parameters, a MIMO time delay system with two outputs and two groups of inputs is considered, i.e., the input vector *u(t)* can be expressed as

$$u = [u_{11}, u_{12}, u_{21}, u_{22}]^T$$

in which the first input group consists of u_{11} and u_{12} and the second input group consists of u_{21} and u_{22}.

Let the transfer function of the reference model be given by

$$W_m(s) = \operatorname{diag}\left\{\frac{1}{s+1}, \frac{1}{s+1}\right\}. \tag{56}$$

The system parameters (55) and reference model (56) satisfy the assumptions defined in section 2 and therefore we can use the controller structure (37) with update rules (43) for this example. Simulation results are obtained for the failure pattern

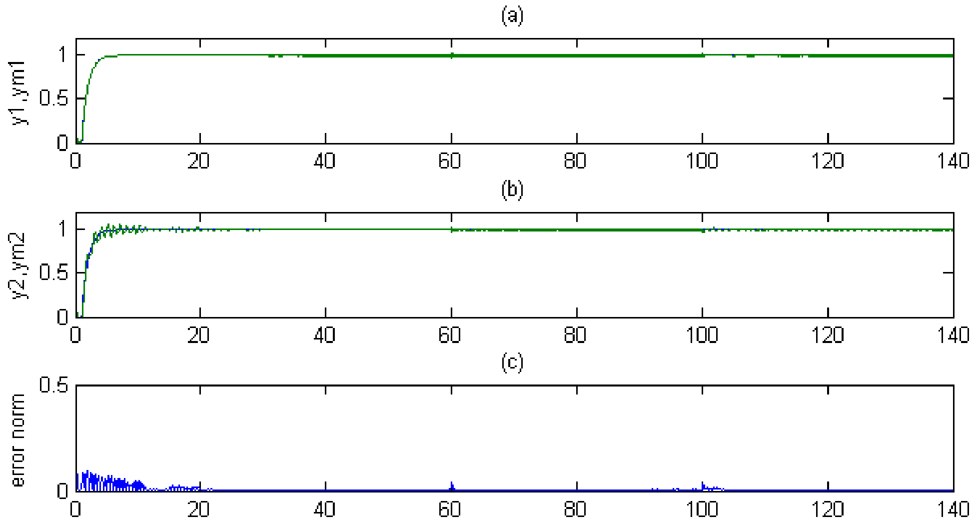

Fig. 1 Simulation results: (a) The plant and reference model outputs and; (b) The plant and reference model outputs and; (c) The error norm

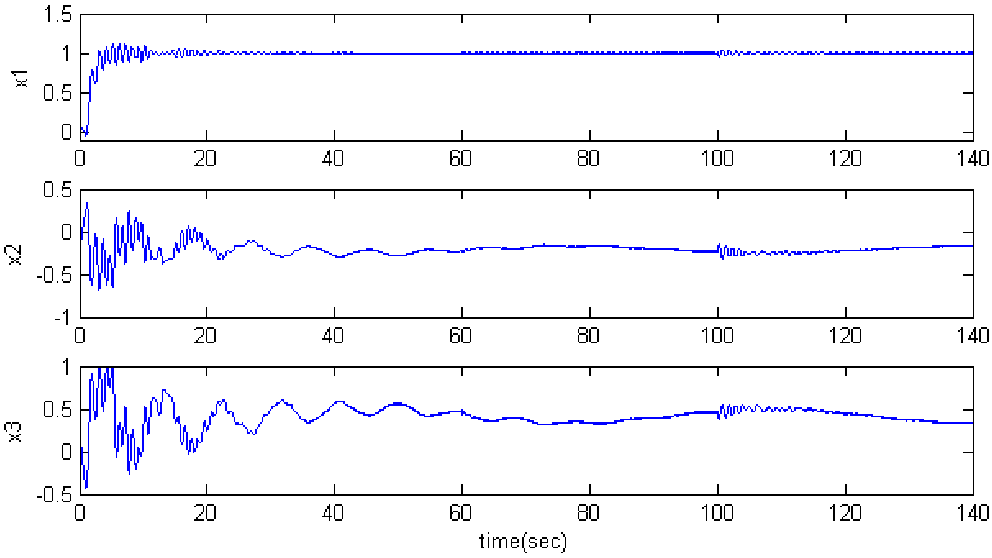

Fig. 2 Simulation results: The system states.

$$u_{12}(t) = -1, \quad t \geq 60,$$
$$u_{21}(t) = 0.5, \quad t \geq 100,$$

in which failures occur in the second actuator in the first group and the first actuator in the second group. All parameters of the system and actuator failures are assumed to be unknown to the controller. The only information available to design the controller is that assumptions (A1) – (A7) are satisfied. Fig. 1 and 2 show the simulation results by choosing the controller parameters $\gamma_1 = 250$, $\gamma_2 = 20$, $\gamma_{I1} = \gamma_{I2} = 0.01$, $\Lambda(s) = (s+1)^2$ and the reference input $r(t) = [1,1]^T$.

It is clear from simulation results that design performances are satisfied. At the time instant when one actuator fails, there exist a transient response in the tracking error, but as the time goes on, the tracking error converges to zero. Clearly the values affect the transient performance of the closed-loop system. Increasing γ_i values will improve the transient performance of the system response and speed up the convergence of to zero. But large values, may make the differential equation of updating the gains, stiff that will require a very small sampling period and therefore, more difficult to solve numerically. Thus, these gains may need to be selected suitably according to the performance that we expect of our system.

7 Conclusions

A Robust output-feedback adaptive actuator failure compensation controller is suggested for MIMO linear systems with unknown time varying state delay. The controlled plant is considered to have M groups of inputs and M outputs. In each actuator group, unknown actuator failures of stuck type may occur. The controller is designed based on the SPR-Lyapunov design method to drive a suitable structure for the case with the relative degree of one. The two component controller structure ensures asymptotic output tracking and robustness with respect to unknown time varying state delay and external disturbances. The results of this paper can be extended to higher relative degrees using normalized MRAC schemes.

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