The Wavelet Transform-Domain LMS Adaptive Filter Algorithm with Variable Step-Size

M. Shams Esfand Abadi*(C.A.), H. Mesgarani* and S. M. Khademiyan*

Abstract: The wavelet transform-domain least-mean square (WTDLMS) algorithm uses the self-orthogonalizing technique to improve the convergence performance of LMS. In WTDLMS algorithm, the trade-off between the steady-state error and the convergence rate is obtained by the fixed step-size. In this paper, the WTDLMS adaptive algorithm with variable step-size (VSS) is established. The step-size in each subfilter changes according to the largest decrease in mean square deviation. The simulation results show that the proposed VSS-WTDLMS has faster convergence rate and lower misadjustment than ordinary WTDLMS.

Keywords: Adaptive Filter, Wavelet Transform Domain LMS (WTDLMS), Variable Step-Size, Mean Square Deviation

1. Introduction

Adaptive filters are widely used in various applications such as system identification, channel equalization, noise cancellation, active noise control, and so on [1], [2]. The most popular adaptive filters are the least mean squares (LMS) and normalized LMS (NLMS) algorithms due to their simplicity. However, these algorithms have slow convergence for colored input signals [3], [4]. To solve this problem, the transform domain adaptive filter (TDAF) algorithms have been proposed [5].

The TDAF algorithms exploit the de-correlation properties of some well-known signal transforms, such as the discrete Fourier transform (DFT) and the discrete cosine transform (DCT), in order to pre-whiten the input data and speed up filter convergence [6], [7], and [8]. In the wavelet transform domain least mean square (WTDLMS) adaptive filtering, the projections of the input signal onto the orthogonal subspaces are used as inputs to a linear combiner. The weights of the linear combiner can hence be updated by the LMS algorithm while normalizing the power at each resolution level to achieve faster and uniform convergence of all weights to the optimal [9], [10].

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In the above mentioned algorithms, the fixed step-size can change the convergence rate and the steady state mean square error (MSE). With optimally selecting the step-size during the adaptation, we obtain fast convergence rate and low steady state mean square error at the same time. In the case of variable step-size (VSS) methods, various approaches have been proposed in the literatures [11], [12]. One of the most important strategy in this issue was pre-

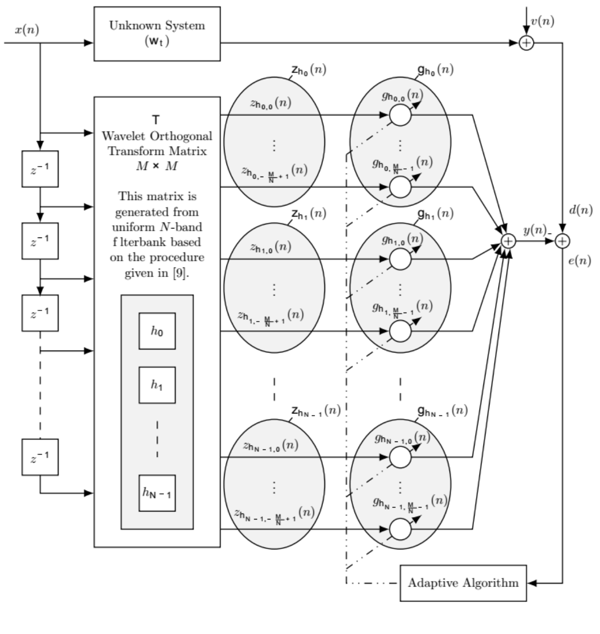

Fig. 1. Structure of the WTDLMS algorithm.

sented in [13]. This approach was successfully extended to the different adaptive filter algorithms in [14]. In this paper, the VSS-WTDLMS is introduced. In the proposed VSS-WTDLMS, the step-size in each subfilter changes according to the largest decrease in mean square deviation. In comparison with ordinary WTDLMS [15], the VSS-WTDLMS has faster convergence speed and lower steady-state MSE.

The reminder of this paper is organized as follows. In Section 2, the WTDLMS algorithm is briefly reviewed. The new VSS-WTDLMS is proposed in Section 3. The computational complexity of the VSS-WTDLMS is discussed in Section 4. Finally, before concluding the paper, the usefulness of this algorithm is demonstrated by presenting several simulation results.

Throughout the paper, represents transpose, takes the squared Euclidean norm, and shows the Expectation.

2. The WTDLMS Adaptive Algorithm

Consider a linear data model for as

$$d(n) = \mathbf{x}^T(n)\mathbf{w}_t + \upsilon(n) \tag{1}$$

where w_t is an unknown M-dimensional vector that we expect to estimate, $\upsilon(n)$ is the measurement noise with variance σ^2_υ, and $\mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-M+1)]^T$ denotes an M-dimensional input (regressor) vector. It is assumed that $\upsilon(n)$ is zero mean, white, Gaussian, and independent of $x(n)$. Fig. 1 shows the structure of the WTDLMS algorithm [9]. In this figure, the MxM matrix T is an orthogonal matrix that is derived from a uniform N-band filter bank with filters denoted by $h_0, h_1, \ldots, h_{N-1}$ following the procedure given in [9]. In matrix form, the orthogonal WT can be expressed as $z(n)=Tx(n)$. This vector can be represented as $\mathbf{z}(n) = [\mathbf{z}^T_{h_0}(n), \mathbf{z}^T_{h_1}(n), \ldots, \mathbf{z}^T_{h_{N-1}}(n)]^T$ where $\mathbf{z}_{h_i}(n)$'s are output vectors of an N-band filter bank. By splitting the $g(n)$ into N subfilters, each having $\frac{M}{N}$ coefficients, $\mathbf{g}(n) = [\mathbf{g}^T_{h_0}(n), \mathbf{g}^T_{h_1}(n), \ldots, \mathbf{g}^T_{h_{N-1}}(n)]^T$, the output signal can be stated as

$$y(n) = \sum_{i=0}^{N-1} \mathbf{g}^T_{h_i}(n)\mathbf{z}_{h_i}(n) \tag{2}$$

and the error signal is obtained by $e(n) = d(n) - y(n)$. The update equation for each subfilter in WTDLMS is given by

$$\mathbf{g}_{h_i}(n+1) = \mathbf{g}_{h_i}(n) + \mu \frac{\mathbf{z}_{h_i}(n)}{\sigma^2_{h_i}(n)} e(n) \tag{3}$$

where $\sigma^2_{h_i}(n)$ can be computed iteratively by

$$\sigma^2_{h_i}(n) = \alpha\sigma^2_{h_i}(n-1) + (1-\alpha)\left\|\mathbf{z}_{h_i}(n)\right\|^2 \tag{4}$$

with a smoothing factor $\alpha\left(0 \ll \alpha < 1\right)$.

3. The VSS-WTDLMS Adaptive Algorithm

By defining the weight error vector $\tilde{\mathbf{g}}_{h_i}(n) = \mathbf{g}^\circ_{h_i} - \mathbf{g}_{h_i}(n)$, where $\mathbf{g}^\circ_{h_i}$ is the true unknown subfilter coefficients, the weight error vector update equation for WTDLMS for each subfilter can be represented as

$$\tilde{\mathbf{g}}_{h_i}(n+1) = \tilde{\mathbf{g}}_{h_i}(n) + \mu_{h_i}(n) \frac{\mathbf{z}_{h_i}(n)}{\sigma^2_{h_i}(n)} e(n) \tag{5}$$

In (5), $\mu_{h_i}(n)$ is a variable step-size in subfilter. Taking the squared Euclidean norm from the both sides of (5) and then the expectation leads to

$$E[\left\|\tilde{\mathbf{g}}_{h_i}(n+1)\right\|^2] = E[\left\|\tilde{\mathbf{g}}_{h_i}(n)\right\|^2] - \Delta \tag{6}$$

where

$$\begin{aligned}\Delta = {} & 2\mu_{h_i}(n)E[\frac{\tilde{\mathbf{g}}^T_{h_i}(n)\mathbf{z}_{h_i}(n)e(n)}{\sigma^2_{h_i}(n)}] \\ & - \mu^2_{h_i}(n)E[\frac{e^2(n)\left\|\mathbf{z}_{h_i}(n)\right\|^2}{\sigma^4_{h_i}(n)}]\end{aligned} \tag{7}$$

Maximizing Δ with respect to $\mu_{h_i}(n)$ leads to the following optimum step-size

$$\mu^\circ_{h_i}(n) = \frac{E[\frac{\tilde{\mathbf{g}}^T_{h_i}(n)\mathbf{z}_{h_i}(n)e(n)}{\sigma^2_{h_i}(n)}]}{E[\frac{e^2(n)\left\|\mathbf{z}_{h_i}(n)\right\|^2}{\sigma^4_{h_i}(n)}]} \tag{8}$$

Since $e_a(n) = \sum_{i=0}^{N-1} \tilde{\mathbf{g}}^T_{h_i}(n)\mathbf{z}_{h_i}(n)$, we use the approximation for a priori error as, $e_a(n) = N\tilde{\mathbf{g}}^T_{h_i}(n)\mathbf{z}_{h_i}(n)$.

Therefore we have

$$\mu^\circ_{h_i}(n) \approx \frac{1}{N} \frac{E[\frac{e^2_a(n)}{\sigma^2_{h_i}(n)}]}{E[\frac{e^2_a(n)\left\|\mathbf{z}_{h_i}(n)\right\|^2}{\sigma^4_{h_i}(n)}] + \sigma^2_\upsilon E[\frac{\left\|\mathbf{z}_{h_i}(n)\right\|^2}{\sigma^4_{h_i}(n)}]} \tag{9}$$

By defining $\mathbf{q}_{h_i}(n) = \frac{\mathbf{z}_{h_i}(n)e_a(n)}{\sigma^2_{h_i}(n)}$, we obtain that $\left\|\mathbf{q}_{h_i}(n)\right\|^2 = \frac{e^2_a(n)\left\|\mathbf{z}_{h_i}(n)\right\|^2}{\sigma^4_{h_i}(n)}$. Then, the optimum step-size is given by

$$\mu^{\circ}_{h_i}(n) \approx \frac{1}{N}\frac{E[\|\mathbf{q}_{h_i}(n)\|^2]}{E[\|\mathbf{q}_{h_i}(n)\|^2]+\sigma_{\upsilon}^2 E[\frac{\|\mathbf{z}_{h_i}(n)\|^2}{\sigma^4_{h_i}(n)}]} \tag{10}$$

Applying the expectation into the leads to the

$$E[\mathbf{q}_{h_i}(n)] = E[\frac{\mathbf{z}_{h_i}(n)e(n)}{\sigma^2_{h_i}(n)}] \tag{11}$$

We propose to estimate $E[\mathbf{q}_{h_i}(n)]$ by time averaging as follows:

$$\hat{\mathbf{q}}_{h_i}(n) = \beta\hat{\mathbf{q}}_{h_i}(n) + (1-\beta)\frac{\mathbf{z}_{h_i}(n)e(n)}{\sigma^2_{h_i}(n)} \tag{12}$$

where $0 \ll \beta < 1$. Using $\|\hat{\mathbf{q}}_{h_i}(n)\|^2$ instead of $E[\|\mathbf{q}_{h_i}(n)\|^2]$, the VSS-WTDLMS for each subfilter is established as

$$\mathbf{g}_{h_i}(n+1) = \mathbf{g}_{h_i}(n) + \mu_{h_i}(n)\frac{\mathbf{z}_{h_i}(n)}{\sigma^2_{h_i}(n)}e(n) \tag{13}$$

where

$$\mu_{h_i}(n) \approx \frac{1}{N}\frac{\|\hat{\mathbf{q}}_{h_i}(n)\|^2}{\|\hat{\mathbf{q}}_{h_i}(n)\|^2 + \frac{\sigma^2_{\upsilon}}{\sigma^2_{h_i}}} \tag{14}$$

The fully update equation for VSS-WTDLMS can be expressed as

$$\mathbf{g}(n+1) = \mathbf{g}(n) + \mathbf{C}(n)\mathbf{z}(n)e(n) \tag{15}$$

Where

$$\mathbf{C}(n) = \begin{pmatrix} \mathbf{C}_{h_0}(n) & 0 & \dots & 0 \\ 0 & \mathbf{C}_{h_1}(n) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \mathbf{C}_{h_{N-1}}(n) \end{pmatrix} \tag{16}$$

and $\mathbf{C}_{h_i}(n) = \frac{\mu_{h_i}(n)}{\sigma^2_{h_i}(n)}.\mathbf{I}$ is the $\frac{M}{N} \times \frac{M}{N}$ matrix. Table 1 summarizes the procedure of the VSS-WTDLMS adaptive algorithm.

4. Computational Complexity

Table 2 describes the computational complexity of the proposed VSS-WTDLMS algorithm. The number of multiplications and divisions have been calculated for each terms. In the following, Table 3 compares the computational complexity of various VSS-TDLMS algorithms. These algorithms are from [7], [11], [12] and [14]. In this Table, M is the number of filter coefficients, N is the number of subbands, M_i is the number of past values of the i^{th} transform coefficient, and L is number of past squared values of the error. The VSS-WTDLMS algorithm needs $M^2+6M+5N$ multiplications and $4N$ divisions. It is interesting to note that using Haar wavelet transform (HWT) leads to the only $6M+5N$ multiplications which is significantly lower than other VSS transform domain adaptive algorithms.

Table 1. The VSS-WTDLMS Adaptive Algorithm.

for $n = 0,1,\dots$

$\mathbf{x}(n) = [x(n), x(n-1), \dots, x(n-M+1)]^T$

$\mathbf{z}(n) = \mathbf{T}\mathbf{x}(n)$

$\mathbf{z}(n) = [\mathbf{z}^T_{h_0}(n), \mathbf{z}^T_{h_1}(n), \dots, \mathbf{z}^T_{h_{N-1}}(n)]^T$

$\mathbf{g}(n) = [\mathbf{g}^T_{h_0}(n), \mathbf{g}^T_{h_1}(n), \dots, \mathbf{g}^T_{h_{N-1}}(n)]^T$

$e(n) = d(n) - \sum_{i=0}^{N-1}\mathbf{g}^T_{h_i}(n)\mathbf{z}_{h_i}(n)$

for $i = 0,1,\dots$

$\sigma^2_{h_i}(n) = \alpha\sigma^2_{h_i}(n-1) + (1-\alpha)\|\mathbf{z}_{h_i}(n)\|^2$

$\hat{\mathbf{q}}_{h_i}(n) = \beta\hat{\mathbf{q}}_{h_i}(n) + (1-\beta)\frac{\mathbf{z}_{h_i}(n)}{\sigma^2_{h_i}(n)}e(n)$

$\mu_{h_i}(n) = \frac{1}{N}\frac{\|\hat{\mathbf{q}}_{h_i}(n)\|^2}{\|\hat{\mathbf{q}}_{h_i}(n)\|^2 + \frac{\sigma^2_{\upsilon}}{\sigma^2_{h_i}}}$

$\mathbf{g}_{h_i}(n+1) = \mathbf{g}_{h_i}(n) + \mu_{h_i}(n)\frac{\mathbf{z}_{h_i}(n)}{\sigma^2_{h_i}(n)}e(n)$

end

end

5. Simulation Results

We demonstrated the performance of the proposed algorithm by several computer simulations in a system identification scenario. The unknown impulse response is randomly selected with 16 taps ($M=16$). The input signal is an AR(1) signal generated by passing a zero-mean white Gaussian noise through a first-order system $H(z) = \frac{1}{1-0.9z^{-1}}$. An additive white Gaussian noise was added to the system output, setting the signal-to-noise ratio (SNR) to 30 dB. The Haar wavelet transform (HWT) was used in all simulations which leads to the reduction of computational complexity due to the elements (+1 and -1) in HWT. The values of α and β were set to 0.995 and 0.9 respectively.

Table 2. The Computational Complexity of VSS-WTDLMS Algorithm

Equation	in subband		Total	
	×	÷	×	÷
$\mathbf{z}(n)=\mathbf{T}\mathbf{x}(n)$	-	-	M^2	-
$e(n)=d(n)-\mathbf{g}^T(n)\mathbf{z}(n)$	-	-	M	-
$\sigma_{h_i}^2(n)=\alpha\sigma_{h_i}^2(n-1)+(1-\alpha)\left\Vert\mathbf{z}_{h_i}(n)\right\Vert^2$	$\frac{M}{N}+2$	-	$M+2N$	-
$\hat{\mathbf{q}}_{h_i}(n)=\beta\hat{\mathbf{q}}_{h_i}(n)+(1-\beta)\frac{\mathbf{z}_{h_i}(n)}{\sigma_{h_i}^2(n)}e(n)$	$2\frac{M}{N}+1$	1	$2M+N$	N
$\mu_{h_i}(n)=\frac{1}{N}\frac{\left\Vert\hat{\mathbf{q}}_{h_i}(n)\right\Vert^2}{\left\Vert\hat{\mathbf{q}}_{h_i}(n)\right\Vert^2+\frac{\sigma_\upsilon^2}{\sigma_{h_i}^2}}$	$\frac{M}{N}+1$	2	$M+N$	$2N$
$\mathbf{g}_{h_i}(n+1)=\mathbf{g}_{h_i}(n)+\mu_{h_i}(n)\frac{\mathbf{z}_{h_i}(n)}{\sigma_{h_i}^2(n)}e(n)$	$\frac{M}{N}+1$	1	$M+N$	N
Total Multiplications : $M^2+6M+5N$				

Table 3. The Computational Complexity of Various VSS-WTLMS

Algorithm	Multiplications	Divisions
DCT-LMS [7]	$M^2+(M_t+4)M+1$	$2M$
VSS-TDLMS [11]	$M^2+5M+L+2$	$M+1$
VSS-TDLMS [12]	M^2+8M+8	$M+1$
VSS-TDLMS [14]	M^2+5M+8	$M+1$
VSS-WTDLMS	$M^2+6M+5N$	$4N$
VSS-WTDLMS (HWT)	$6M+5N$	$4N$

Table 4. The Parameters In VSS-TDLMS and VSS-WTDLMS Algorithms.

DCT-LMS [7]	VSS-TDLMS [11]
$\beta=0.9985$ $\gamma=8\times10^{-3}$ $M_t=10$	$\alpha=0.99$ $\beta=0.9$ $\gamma=10^{-3}$ $L=10$ $\mu_{max}=5\times10^{-2}$ $\mu_{min}=4.7\times10^{-3}$
VSS-TDLMS [12]	**VSS-TDLMS [14]**
$\beta=0.98$ $\gamma=0.98$	$\alpha=0.995$ $\beta=0.9$ $\gamma=0.9$ $\mu_{max}=0.5$ $C=M\times\sigma_\upsilon^2$
VSS-WTDLMS	
$\alpha=0.995$ $\beta=0.9$	

Figs. 2-4 show the mean square deviation (MSD) learning curves of proposed VSS-WTDLMS and ordinary WT-DLMS algorithm for different values of *N*. In WTDLMS, different values for the step-size have been selected. We observe that VSS-WTDLMS has faster convergence speed and lower steady-state error than ordinary WT-DLMS algorithm for all values of *N*. The comparison of VSS-WTDLMS with recently and famous VSS-TDLMS algorithms has been presented in Fig. 5 [7], [11], [12] and [14]. The parameters of the simulated algorithms have been chosen according to the Table 4. This figure shows that, the proposed VSS-WTDLMS has better convergence speed an lower steady-state error than other VSS-TDLMS algorithms for all values of *N*. Also, the computational complexity of the proposed algorithm is lower than other

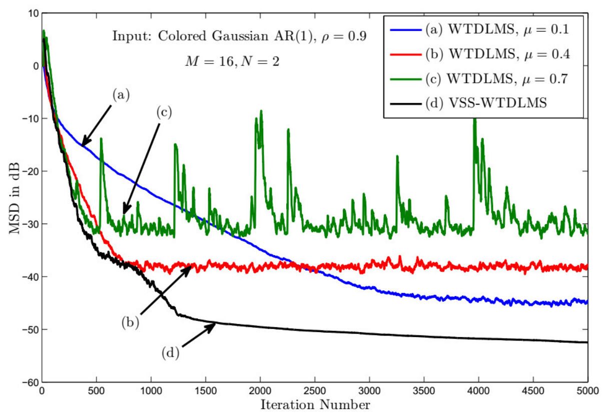

Fig. 2. The MSD learning curves of WTDLMS and VSS-WT-DLMS algorithms with N = 2.

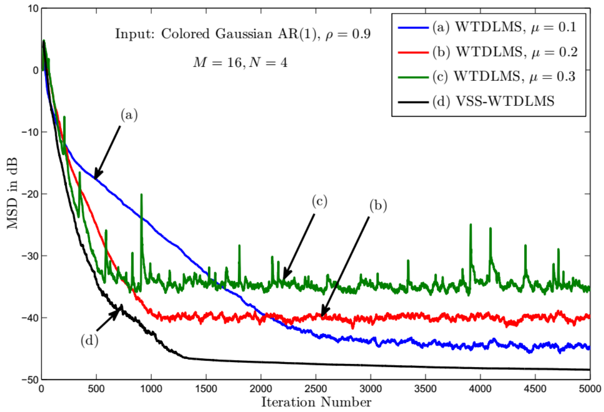

Fig. 3. The MSD learning curves of WTDLMS and VSS-WT-DLMS algorithms with N = 4.

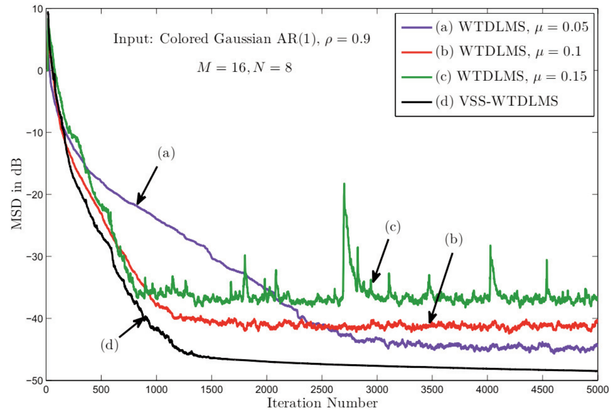

Fig. 4. The MSD learning curves of WTDLMS and VSS-WT-DLMS algorithms with N = 8.

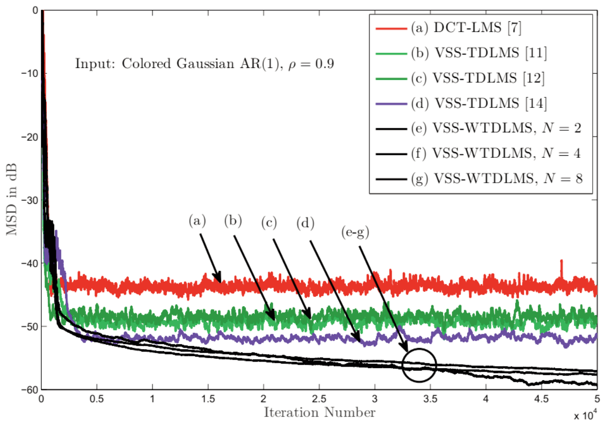

Fig. 5. The MSD learning curves of various VSS-TDLMS and VSS-WTDLMS algorithms.

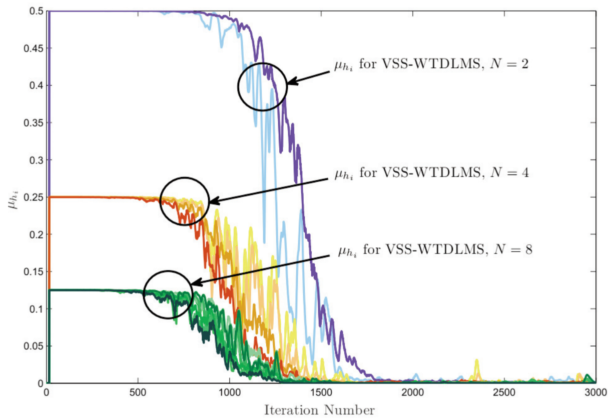

Fig. 6. Variation of the step-size in each subband during the adaptation for VSS-WTDLMS.

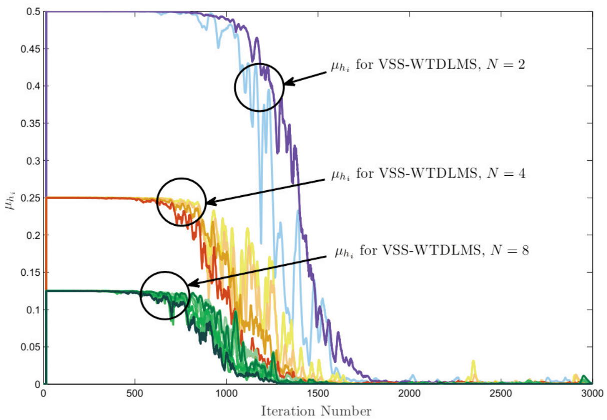

Fig. 7. The number of filter coefficients versus the filter length (M) in various VSS-TDLMS, WTDLMS, and VSS-WTDLMS algorithms.

VSS-TDLMS algorithms due to using HWT.

Fig. 6 presents the variation of the step-size in each sub-band for VSS-WTDLMS algorithm during the adaptation. As we can see, the step-sizes start from large values and end with low values for N=*2, 4* , and 8. Finally, Fig. 7 shows the number of filter coefficients versus the filter length for VSS-TDLMS, WTDLMS, and VSS-WTDLMS algorithms. This figure indicates that the computational complexity of VSS-WTDLMS with HWT is significantly lower than other algorithms.

6. Conclusion

In this paper, the WTDLMS with VSS was established. The step-size in each subband changes according to the largest decrease in mean square deviation. The simulation results indicated that the proposed VSS-WTDLMS had faster convergence speed and lower steady-state error than ordinary WTDLMS and other VSS-TDLMS algorithms. Also, the computational complexity of VSS-WTDLMS with HWT was significantly lower than other algorithms.

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