Accurate Analysis of Dielectric Backed Planar Conducting Layers of Arbitrarily Shaped in a Rectangular Waveguide

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Abstract: The characteristics of dielectric backed planar conducting layers of arbitrarily shaped in a rectangular waveguide are calculated by means of coupled integral equation technique (CIET) which accurately takes higher order mode interactions. Spectral dyadic green's functions are derived for these equivalent structures. A coupled magnetic filed integral equation formulation is proposed which is solved using method of moment (MoM). Then one matrix equation that involves the all magnetic currents of non metallic parts (aperture parts) of discontinuities is derived. This single matrix formulation replaces the procedure of cascading individual GSM's of each block. By dividing the area of cross section in discontinuities into a given number of subsections, it is possible to model any shape of metallic parts. The proposed technique permits modeling of a variety of structures such as cavity-backed micro strip antenna, frequency selective surfaces (FSS's), waveguide filters with printed irises and generally multilayered media with printed circuits embedded between dielectric layers in a waveguide.
The usefulness of the proposed method and its performance are verified by calculating and simulating of a given structure.

Keywords: Dielectric Backed Planar Conducting Layers, Rectangular Waveguide, Coupled Integral Equation Technique, Method of Moment.

1 Introduction

Many waveguide based microwave and millimeter wave systems are constructed of dielectric backed planar conducting layers of arbitrarily shaped which is located transversely in a waveguide. A wide range of practical structure such as waveguide-based filters, periodic grid structures in waveguides, waveguide-based strip and micro strip filters, patch arrays, densely packaged passive elements and waveguide-based spatial power combiners are among the structures that can be categorized as planar conductive layers in a guided wave environment. All of mentioned structures consist of a number of patterned dielectric backed conducting layers separated by waveguide sections. One way to electromagnetically characterize these structures is using of multilayer method of moments (MoM) technique which results in a very large matrix problem. However generalized scattering matrix (GSM) approach is a good solution. In the GSM method each block

represented by a matrix that relates the coefficients of forward and backward propagating waveguide modes at the two sides of each block. These matrices are cascaded to obtain overall response of whole structure which is cascaded of a number of blocks [1-2]. In GSM approach all propagating and evanescent TE and TM modes are be considered that provide an accurate simulation of interactions between adjacent blocks. A problem occurs when the separation between these adjacent blocks becomes electrically small. This necessitates a significant increase in the number of waveguide modes which in turn results in the size of GSM and subsequently increases the memory and computational requirements. By now much effort have been done to reduce the size of microwave devices, such as compact bandpass filter utilizing dielectric filled waveguides [3-4] and bandpass filters using frequency selective surfaces [5-6]. In all of theses miniaturized structures separation between neighboring dielectric backed conducting layers are small compared to length wave. The solution is to model the whole closely spaced layers as a single block rather than as the cascade of a number of blocks. In [7] coupled integral equation technique have been applied for the analysis of H-plane

wave guide filters in which the interaction between all discontinuities are accurately described.

In this paper a full wave integral equation technique is developed for electromagnetically modeling of structures in which dielectric backed planar conducting layers of arbitrarily shaped are located transversely in a waveguide. A coupled set of magnetic integral equations have been applied for unknown magnetic current densities at the nonmetal parts (aperture parts) of the interfaces. These equations is solved by method of moment and using piecewise triangular or piecewise sinusoidal overlapping basis and testing functions for the magnetic current densities.

## 2 Coupled Integral Equation Technique Formulation

Consider a rectangular waveguide shown in Fig. 1 with three dielectric backed planar conducting layers. Arbitrarily shaped metallization S_{m1}, S_{m2} and S_{m3} are located on the interfaces 1, 2 and 3 respectively. Relative permittivity of dielectric layers are $\varepsilon_{r1}, \varepsilon_{r2}$ and ε_{r3} with thickness d_1, d_2 and d_3 respectively. The loss of conductors and dielectrics has not been considered for the simplicity. According to equivalence theorem the electric and magnetic field in the region $z < 0$ is the total of incident electric and magnetic field (direct from an impressed current source and reflected from the interface 1 when replaced by a perfectly conducting plane) and a scattered (reflected) field generated by an induced magnetic current on the non metallization parts (aperture parts) of interface 1. Field in the regions $0 < z < \ell_1$ and $\ell_1 < z < \ell_1 + \ell_2$ is only due to induced magnetic current on the aperture parts of interfaces 1, 2 and interfaces 2, 3 respectively. In the same way the electric and magnetic field in the region $z > \ell_1 + \ell_2$ is due to induced magnetic current on the aperture parts of interface 3 only, but the continuity of the magnetic fields across the interfaces provides the interaction of all regions an necessitates the formulation of the problem in terms of coupled magnetic integral equations.

Fig. 2 shows the equivalent structure for obtaining fields in varies regions. Aperture parts of interface 1 (located at the $z=0$ plane) is replaced by a perfectly conducting plane (shorted aperture), with the original tangential electric field at the aperture parts restored at $z = 0^+$ and $z = 0^-$ by appropriate magnetic surface currents $\mathbf{M}_1$ and $-\mathbf{M}_1$ ($\mathbf{M}_1$ in the reverse direction), respectively.

Aperture parts of interface 2 located at the $z = \ell_1$ plane is replaced by a perfectly conducting plane (shorted aperture), with the original tangential electric field at the aperture parts restored at $z = \ell_1^+$ and $z = \ell_1^-$ by appropriate magnetic surface currents $\mathbf{M}_2$ and $-\mathbf{M}_2$, respectively.
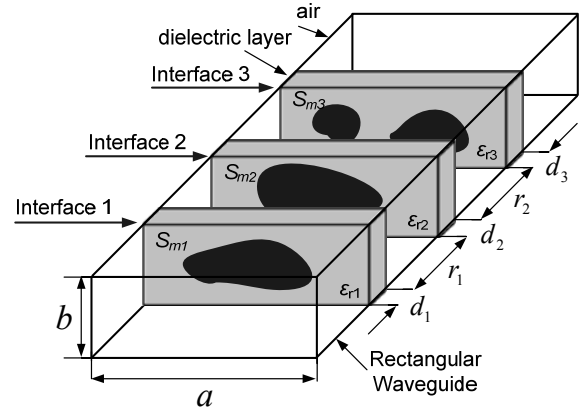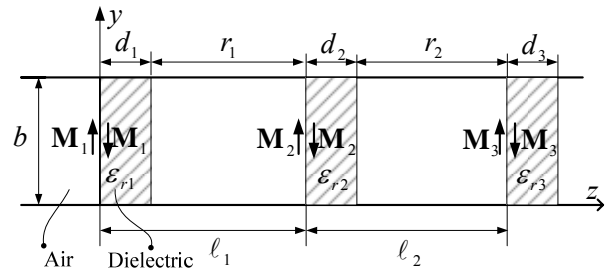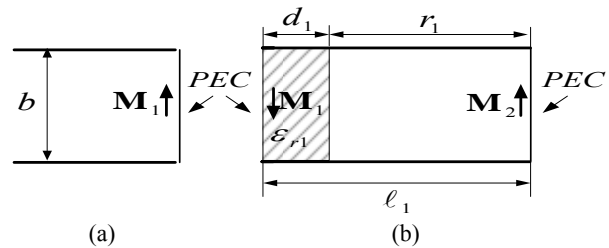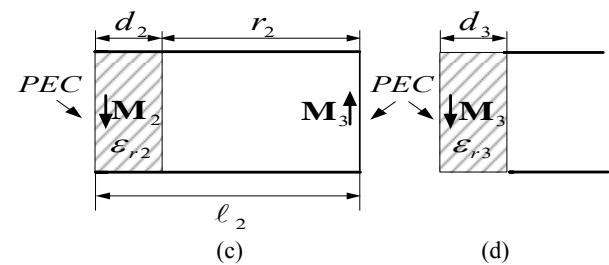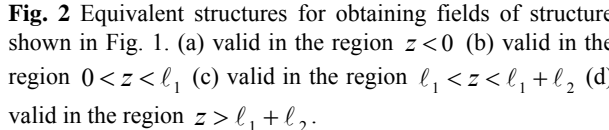

Fig. 1 Geometry of a rectangular waveguide consisting dielectric backed planar conducting layers of arbitrarily shaped. (a) Three dimension view. (b) Side view with equivalent magnetic current.

Fig. 2 Equivalent structures for obtaining fields of structure shown in Fig. 1. (a) valid in the region $z < 0$ (b) valid in the region $0 < z < \ell_1$ (c) valid in the region $\ell_1 < z < \ell_1 + \ell_2$ (d) valid in the region $z > \ell_1 + \ell_2$.
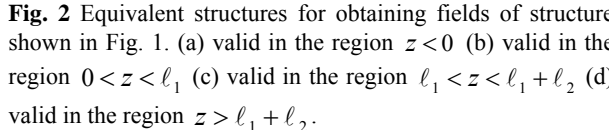
Similarly $\mathbf{M}_3$ and $-\mathbf{M}_3$ can be replaced by a perfectly conducting plane at $z = (\ell_1 + \ell_2)^-$ and $z = (\ell_1 + \ell_2)^+$, respectively. As shown in Fig. 3(a) the scattered fields at $z = 0^-$ is radiated by $\mathbf{M}_1$ in the

presence of the conducting plane located at $z = 0$ and the environment in the region $z < 0$. The total fields at $z = 0^+$ and $z = \ell_1^-$ is radiated by $-\mathbf{M}_1$ and $\mathbf{M}_2$ in the presence of two conducting planes, located at $z = 0$ and $z = \ell_1$ and the environment in the region $0 < z < \ell_1$. In the same way the total fields at $z = \ell_1^+$ and $z = (\ell_1 + \ell_2)^-$ is radiated by $-\mathbf{M}_2$ and $\mathbf{M}_3$ in the presence of two conducting planes, located at $z = \ell_1$ and $z = \ell_1 + \ell_2$ and the environment in the region $\ell_1 < z < \ell_1 + \ell_2$. Finally the total fields at $z = (\ell_1 + \ell_2)^+$ is radiated by $-\mathbf{M}_3$ in the presence of conducting plane, located at $z = \ell_1 + \ell_2$ and the environment in the region $z > \ell_1 + \ell_2$. By using the conventional spectral domain immittance approach [8-9] one can drive the appropriate spectral domain Green's functions for these equivalent structures. The enforcement of continuity of transverse magnetic fields across the aperture parts provides the interaction of all regions. A coupled set of magnetic field integral equations is obtained by enforcing tangential component of magnetic fields on the aperture part of interfaces. In the following, the final expressions for the magnetic fields are given and the detail of spectral domain immittance approach is omitted. With all the appropriate variables defined in the Appendix, tangential magnetic fields at $z = 0$, $z = \ell_1$ and $z = \ell_1 + \ell_2$, approaching from either side of the plane, take the following forms. Quantities with a tilde (~) are Fourier transforms of corresponding quantities without tilde.

$$\begin{bmatrix} \widetilde{H}_x \\ \widetilde{H}_y \end{bmatrix}_{z=0^-} = \begin{bmatrix} \widetilde{G}^1_{xx} & \widetilde{G}^1_{xy} \\ \widetilde{G}^1_{yx} & \widetilde{G}^1_{yy} \end{bmatrix} \begin{bmatrix} \widetilde{M}_{1x} \\ \widetilde{M}_{1y} \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \widetilde{H}_x \\ \widetilde{H}_y \end{bmatrix}_{z=0^+} = -\begin{bmatrix} \widetilde{G}^2_{xx} & \widetilde{G}^2_{xy} \\ \widetilde{G}^2_{yx} & \widetilde{G}^2_{yy} \end{bmatrix} \begin{bmatrix} \widetilde{M}_{1x} \\ \widetilde{M}_{1y} \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} \widetilde{H}^{sc}_x \\ \widetilde{H}^{sc}_x \end{bmatrix}_{z=0^+} = -\begin{bmatrix} \widetilde{G}^3_{xx} & \widetilde{G}^3_{xy} \\ \widetilde{G}^3_{yx} & \widetilde{G}^3_{yy} \end{bmatrix} \begin{bmatrix} \widetilde{M}_{2x} \\ \widetilde{M}_{2y} \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} \widetilde{H}_x \\ \widetilde{H}_y \end{bmatrix}_{z=\ell_1^-} = \begin{bmatrix} \widetilde{G}^4_{xx} & \widetilde{G}^4_{xy} \\ \widetilde{G}^4_{yx} & \widetilde{G}^4_{yy} \end{bmatrix} \begin{bmatrix} \widetilde{M}_{2x} \\ \widetilde{M}_{2y} \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} \widetilde{H}^{sc}_x \\ \widetilde{H}^{sc}_x \end{bmatrix}_{z=\ell_1^-} = \begin{bmatrix} \widetilde{G}^5_{xx} & \widetilde{G}^5_{xy} \\ \widetilde{G}^5_{yx} & \widetilde{G}^5_{yy} \end{bmatrix} \begin{bmatrix} \widetilde{M}_{1x} \\ \widetilde{M}_{1y} \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} \widetilde{H}_x \\ \widetilde{H}_y \end{bmatrix}_{z=\ell_1^+} = -\begin{bmatrix} \widetilde{G}^6_{xx} & \widetilde{G}^6_{xy} \\ \widetilde{G}^6_{yx} & \widetilde{G}^6_{yy} \end{bmatrix} \begin{bmatrix} \widetilde{M}_{2x} \\ \widetilde{M}_{2y} \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} \widetilde{H}^{sc}_x \\ \widetilde{H}^{sc}_x \end{bmatrix}_{z=\ell_1^+} = -\begin{bmatrix} \widetilde{G}^7_{xx} & \widetilde{G}^7_{xy} \\ \widetilde{G}^7_{yx} & \widetilde{G}^7_{yy} \end{bmatrix} \begin{bmatrix} \widetilde{M}_{3x} \\ \widetilde{M}_{3y} \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} \widetilde{H}_x \\ \widetilde{H}_y \end{bmatrix}_{z=(\ell_1+\ell_2)^-} = \begin{bmatrix} \widetilde{G}^8_{xx} & \widetilde{G}^8_{xy} \\ \widetilde{G}^8_{yx} & \widetilde{G}^8_{yy} \end{bmatrix} \begin{bmatrix} \widetilde{M}_{3x} \\ \widetilde{M}_{3y} \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} \widetilde{H}^{sc}_x \\ \widetilde{H}^{sc}_x \end{bmatrix}_{z=(\ell_1+\ell_2)^-} = \begin{bmatrix} \widetilde{G}^9_{xx} & \widetilde{G}^9_{xy} \\ \widetilde{G}^9_{yx} & \widetilde{G}^9_{yy} \end{bmatrix} \begin{bmatrix} \widetilde{M}_{2x} \\ \widetilde{M}_{2y} \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} \widetilde{H}_x \\ \widetilde{H}_y \end{bmatrix}_{z=(\ell_1+\ell_2)^+} = -\begin{bmatrix} \widetilde{G}^{10}_{xx} & \widetilde{G}^{10}_{xy} \\ \widetilde{G}^{10}_{yx} & \widetilde{G}^{10}_{yy} \end{bmatrix} \begin{bmatrix} \widetilde{M}_{3x} \\ \widetilde{M}_{3y} \end{bmatrix} \tag{10}$$

where

$$\widetilde{G}^i_{xx} = Y^{ih} \sin^2 \theta + Y^{ie} \cos^2 \theta \tag{11}$$

$$\widetilde{G}^i_{xy} = (Y^{ih} - Y^{ie}) \sin \theta \cos \theta \tag{12}$$

$$\widetilde{G}^i_{yx} = \widetilde{G}^i_{xy} \tag{13}$$

$$\widetilde{G}^i_{yy} = Y^{ih} \cos^2 \theta + Y^{ie} \sin^2 \theta \tag{14}$$

for $i = 1, 2, \ldots, 10$.

After enforcing continuity of the tangential magnetic field across the aperture parts of interfaces at $z = 0$, $z = \ell_1$ and $z = \ell_1 + \ell_2$ in the spatial domain, we obtain:

$$\begin{bmatrix} H_{inc,x} \\ H_{inc,y} \end{bmatrix} + \begin{bmatrix} H_x \\ H_y \end{bmatrix}_{z=0^-} = \begin{bmatrix} H_x \\ H_y \end{bmatrix}_{z=0^+} + \begin{bmatrix} H^{SC}_x \\ H^{SC}_y \end{bmatrix}_{z=0^+} \tag{15}$$

$$\begin{bmatrix} H_x \\ H_y \end{bmatrix}_{z=\ell_1^-} + \begin{bmatrix} H^{SC}_x \\ H^{SC}_y \end{bmatrix}_{z=\ell_1^-} = \begin{bmatrix} H_x \\ H_y \end{bmatrix}_{z=\ell_1^+} + \begin{bmatrix} H^{SC}_x \\ H^{SC}_y \end{bmatrix}_{z=\ell_1^+} \tag{16}$$

$$\begin{bmatrix} H_x \\ H_y \end{bmatrix}_{z=(\ell_1+\ell_2)^-} + \begin{bmatrix} H^{SC}_x \\ H^{SC}_y \end{bmatrix}_{z=(\ell_1+\ell_2)^-} = \begin{bmatrix} H_x \\ H_y \end{bmatrix}_{z=(\ell_1+\ell_2)^+} \tag{17}$$

By expressing the fields in term of their spectral green's functions, the matrix equation can be obtained. To solve these set of coupled integral equations (CIE's), we expand magnetic currents in a series of basis functions using traditional x-and y-directed piece-wise sinusoidal or roof-top basis and testing functions. Then apply Galerkin's method to each one of them. Let $B^x_j(x)$ and $B^y_j(y)$ denote the j-th element of a set of basis functions for the functions $M_{ix}(x, y)$ and $M_{iy}(x, y)$, respectively. We have

$$M_{ix}(x, y) = \sum_{j=1}^{N_{ix}} C_{ijx} B^{ix}_j(x) \tag{18}$$

$$M_{iy}(x, y) = \sum_{j=1}^{N_{iy}} C_{ijy} B_j^{iy}(y) \qquad (19)$$

for $i = 1$, 2 and 3 and also $B_j^x(x)$ and $B_j^y(y)$ can be piecewise sinusoidal basis function or piecewise triangular (roof-top) basis functions. At here they are locally roof-top basis functions defined as

$$B_j^{ix}(x) = \begin{cases} \left|1 - \dfrac{x - x_j^i}{c}\right|, & |x - x_j^i| < c_i \quad \& \quad |y - y_j^i| < d_i/2 \\ \text{otherwise}, & 0 \end{cases}$$

$$(20)$$

$$B_j^{iy}(y) = \begin{cases} \left|1 - \dfrac{y - y_j^i}{d}\right|, & |y - y_j^i| < d_i \quad \& \quad |x - y_j^i| < c_i/2 \\ \text{otherwise}, & 0 \end{cases}$$

$$(21)$$

As an example, rectangular cells with the x-directed overlapping roof-top basis functions are shown in Fig. 3. If these expansion are used in the integral equations and Galerkin's method is applied to each one of them, a set of coupled line equations in the expansion coefficients C_{ijx} and C_{ijy} results. The three linear equations can be put in the following form

$$\mathbf{A}_1 \mathbf{C}_1 + \mathbf{A}_2 \mathbf{C}_2 = \mathbf{U} \qquad (22)$$

$$\mathbf{A}_3 \mathbf{C}_1 + \mathbf{A}_4 \mathbf{C}_2 + \mathbf{A}_5 \mathbf{C}_3 = \mathbf{0} \qquad (23)$$

$$\mathbf{A}_6 \mathbf{C}_2 + \mathbf{A}_7 \mathbf{C}_3 = \mathbf{0} \qquad (24)$$

Each matrix in (22)-(24) has been made of 4 or 2 sub matrices. It is notable that some of these matrices may be equal in symmetrical conditions, a fact which, is used to reduce the computational effort. Also it is worth noting that (22)-(24) follow a clear pattern which allows the number of dielectric backed conducting layers to be varied relatively simple.

2 Examples and Numerical Results

As an example a resonant patch array supported by dielectric slab in a rectangular waveguide (Fig. 4) which is selected from [10] is analyzed for application in high frequency EM and quasi-optical transmitting and receiving systems. This design has been performed using a rectangular waveguide whose dimensions are $a = 10.287$ mm, $b = 22.86$ mm and a single dielectric backed conducting layer. The structure can be model by considering $a = 22.86$ mm, $b = 10.287$ mm, $d1 = 25$ mm, $d2 = d3 = 0$ and $\varepsilon_r = 2.33$ in the Fig. 1. The magnitude of S_{11} and S_{21} have been shown in Fig. 5 and compared with reference [1] and HFSS [10] results. There is a good agreement between them.

In the second example a compact bandpass filter is analyzed which is designed by the authors. Fig. 6(a) shows this bandpass filter which has been designed using dielectric backed patterned conducting planes (PCPs) which are located transversely in distinct separations in a WR-90 rectangular waveguide.

For a 3-order chebyshev type bandpass filter with center frequency $f_0 = 10$GHz, the relative bandwidth 10 percent and equal ripples 0.5 dB, the geometry of each patterned conducting plane has been obtained using genetic algorithm. Fig. 6(b) shows the shapes of these patterned conducting planes in which blank pixels correspond to nonmetal part and hatched pixels correspond to metal parts of interface planes. The desired filter can be realized by choosing, $\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_{r3} = 2$ for relative dielectric permittivity, $r_1 = r_2 = 0$, $d_3 = 0$ and $d_1 = d_2 = \ell_1 = \ell_2 = 29.9 \approx 3$ mm.

Fig. 7 shows the transmission characteristic of transmission the filter which is compared by HFSS results. There is a good agreement between proposed method and HFSS results.

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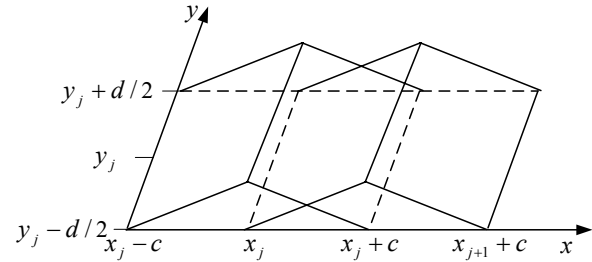

Fig. 3. Rectangular cells with the x-directed overlapping piecewise roof-top basis functions.
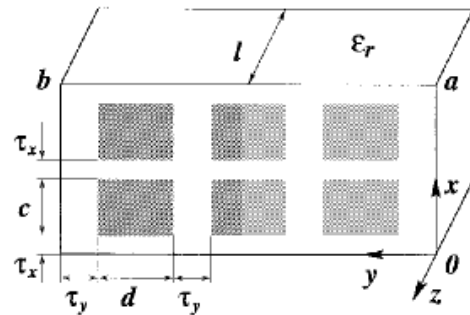

Fig. 4. Geometry of patch array supported by dielectric slab in a rectangular waveguide. $a = 10.287$ mm, $b = 22.86$ mm, $\ell = 25$ mm, $\varepsilon_r = 2.33$, $d = 4.572$ mm, $c = 3.429$ mm, $\tau_x = 1.143$ mm and $\tau_y = 2.286$ mm [1].

APPENDIX

The variables that appear in the spectral dyadic Green's functions in (1) to (14) are given as
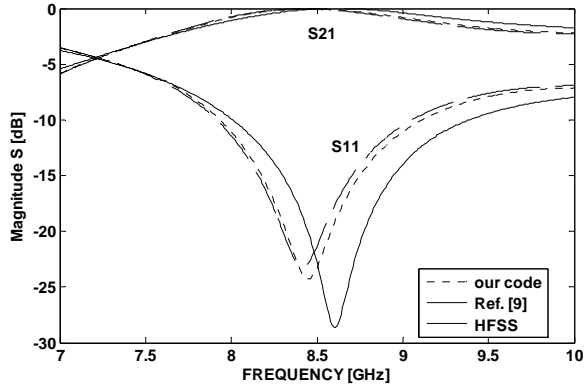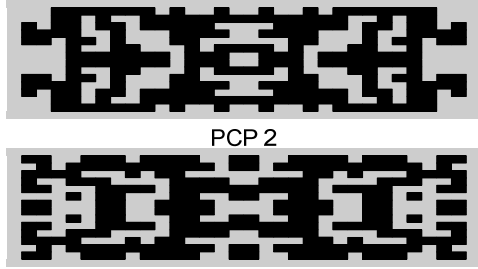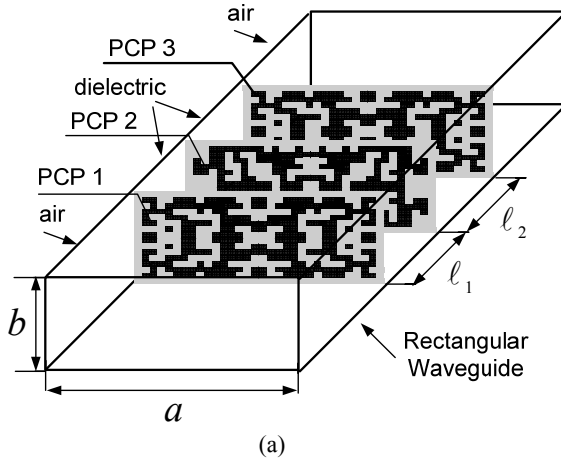
$$Y^{1e,h} = Y_a^{\text{TM,TE}} \qquad (A1)$$

Fig 5. Magnitude of S_{11} and S_{21} for the patch array embedded in a rectangular waveguide.

Fig. 7. Transmission characteristic of the filter

(a)

PCP 2

PCPs 1 and 3

(b)

Fig. 6. (a) Waveguide bandpass filter using patterned conducting planes (PCPs). (b) Designed PCPs

4. Conclusion

Dielectric backed planar conducting layers of arbitrarily shaped in a rectangular waveguide have been analyzed. The analysis method is based of coupled integral equation technique which accurately takes higher order mode interactions. The exact analytical formulas and equations have been extracted which can be used in a variety of structures. The proposed method follow a clear pattern which allows the number of dielectric backed conducting layers of arbitrarily shaped to be varied relatively simple.
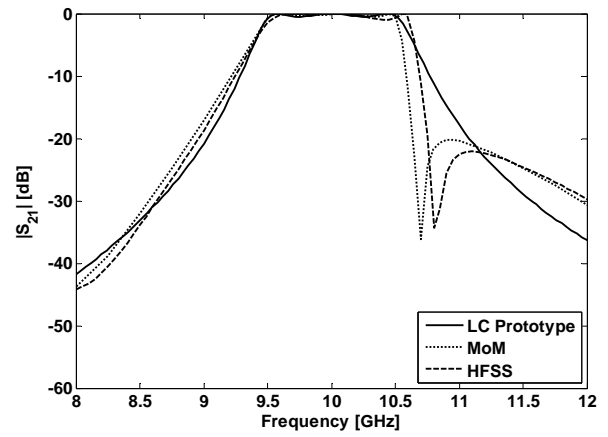
$$Y^{2e,h} = Y_{d1}^{\mathrm{TM,TE}} \frac{Y_a^{\mathrm{TM,TE}} + Y_{d1}^{\mathrm{TM,TE}} \tanh(\gamma_{d1}d_1)\tanh(\gamma_a r_1)}{Y_{d1}^{\mathrm{TM,TE}} \tanh(\gamma_a r_1) + Y_a^{\mathrm{TM,TE}} \tanh(\gamma_{d1}d_1)} \tag{A2}$$

$$Y^{3e,h} = Y_a^{\mathrm{TM,TE}} Y_{d1}^{\mathrm{TM,TE}} \frac{1}{\cosh(\gamma_{d1}d_1)\cosh(\gamma_a r_1)} \tag{A3}$$

$$\times \frac{1}{Y_{d1}^{\mathrm{TM,TE}} \tanh(\gamma_a r_1) + Y_a^{\mathrm{TM,TE}} \tanh(\gamma_{d1}d_1)}$$

$$Y^{4e,h} = Y_a^{\mathrm{TM,TE}} \frac{Y_{d1}^{\mathrm{TM,TE}} + Y_a^{\mathrm{TM,TE}} \tanh(\gamma_{d1}d_1)\tanh(\gamma_a r_1)}{Y_a^{\mathrm{TM,TE}} \tanh(\gamma_{d1}d_1) + Y_{d1}^{\mathrm{TM,TE}} \tanh(\gamma_a r_1)} \tag{A4}$$

$$Y^{5e,h} = Y^{3e,h} \tag{A5}$$

$$Y^{6e,h} = Y_{d2}^{\mathrm{TM,TE}} \frac{Y_a^{\mathrm{TM,TE}} + Y_{d2}^{\mathrm{TM,TE}} \tanh(\gamma_{d2}d_2)\tanh(\gamma_a r_2)}{Y_{d2}^{\mathrm{TM,TE}} \tanh(\gamma_a r_2) + Y_a^{\mathrm{TM,TE}} \tanh(\gamma_{d2}d_2)} \tag{A6}$$

$$Y^{7e,h} = Y_a^{\mathrm{TM,TE}} Y_{d2}^{\mathrm{TM,TE}} \frac{1}{\cosh(\gamma_{d2}d_2)\cosh(\gamma_a r_2)} \tag{A7}$$

$$\times \frac{1}{Y_{d2}^{\mathrm{TM,TE}} \tanh(\gamma_a r_2) + Y_a^{\mathrm{TM,TE}} \tanh(\gamma_{d2}d_2)}$$

$$Y^{8e,h} = Y_a^{\mathrm{TM,TE}} \frac{Y_{d2}^{\mathrm{TM,TE}} + Y_a^{\mathrm{TM,TE}} \tanh(\gamma_{d2}d_2)\tanh(\gamma_a r_2)}{Y_a^{\mathrm{TM,TE}} \tanh(\gamma_{d2}d_2) + Y_{d2}^{\mathrm{TM,TE}} \tanh(\gamma_a r_2)} \tag{A8}$$

$$Y^{9e,h} = Y^{7e,h} \tag{A9}$$

$$Y^{10e,h} = Y_{d3}^{\mathrm{TM,TE}} \frac{Y_a^{\mathrm{TM,TE}} + Y_{d3}^{\mathrm{TM,TE}} \tanh(\gamma_{d3}d_3)}{Y_{d3}^{\mathrm{TM,TE}} + Y_a^{\mathrm{TM,TE}} \tanh(\gamma_{d3}d_3)} \tag{A10}$$

where

$$Y_{di}^{\mathrm{TE}} = \frac{\gamma_{di}}{j\omega\mu_0}, \qquad i = 1,2,3. \tag{A11}$$

$$Y_{di}^{\mathrm{TM}} = \frac{j\omega\varepsilon_0\varepsilon_{ri}}{\gamma_{di}}, \qquad i = 1,2,3. \tag{A12}$$

$$Y_a^{\mathrm{TE}} = \frac{\gamma_a}{j\omega\mu_0} \tag{A13}$$

$$Y_a^{\mathrm{TM}} = \frac{j\omega\varepsilon_0}{\gamma_a} \qquad (A14)$$

$$\gamma_a = \sqrt{k_x^2 + k_y^2 - \omega^2\mu_0\varepsilon_0} \qquad (A15)$$

$$\gamma_{di} = \sqrt{k_x^2 + k_y^2 - \omega^2\mu_0\varepsilon_0\varepsilon_{ri}}, \quad i=1,2,3. \qquad (A16)$$

$$\sin\theta = \frac{k_x}{\sqrt{k_x^2 + k_y^2}} \qquad (A17)$$

$$\cos\theta = \frac{k_y}{\sqrt{k_x^2 + k_y^2}} \qquad (A18)$$

in which $k_x = m\pi/a, \;\; k_y = n\pi/b$ $\qquad$ (A19)

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